ECONSTOR
Make Your Publications Visible.

A Service of
ZBW Leibniz-Informationszentrum Wirtschaft
Leibniz Information Centre for Economics

Výrost, Tomas; Lyócsa, Štefan; Baumöhl, Eduard

**Working Paper**

# Network-based asset allocation strategies

Suggested Citation: Výrost, Tomas; Lyócsa, Štefan; Baumöhl, Eduard (2018) : Network-based asset allocation strategies, ZBW – Leibniz Information Centre for Economics, Kiel, Hamburg

This Version is available at:
http://hdl.handle.net/10419/180063

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

WWW.ECONSTOR.EU

Mitglied der Leibniz-Gemeinschaft

# Network-based asset allocation strategies

Tomas Výrost[a*], Štefan Lyócsa[b,c], Eduard Baumöhl[a,d]

**Abstract**

In this study, we construct financial networks in which nodes are represented by assets and where edges are based on long-run correlations. We construct four networks (complete graph, a minimum spanning tree, a planar maximally filtered graph, and a threshold significance graph) and use three centrality measures (betweenness, eigenvalue centrality, and the expected force). To improve risk-return characteristics of well-known return maximization and risk minimization benchmark portfolios, we propose simple adjustments to portfolio selection strategies that utilize centralization measures from financial networks. From a sample of 45 assets (stock market indices, bond and money market instruments, commodities, and foreign exchange rates) and from data for 1999 to 2015, we show that irrespective of the network and centrality employed, the proposed network-based asset allocation strategies improve key portfolio return characteristics in an out-of-sample framework, most notably, risk and left-tail risk-adjusted returns. Resolving portfolio model selection uncertainties further improves risk-return characteristics. Improvements made to portfolio strategies based on risk minimization are also robust to transaction costs.

**Keywords:** networks; portfolio; centrality; risk-return profile

**JEL Classification:** G10, G11, G15, C61

[a] University of Economics in Bratislava, Economics and Management Institute, Dolnozemská cesta 1, 852 35 Bratislava, Slovakia.

[b] University of Economics in Bratislava, Faculty of National Economy, Dolnozemská cesta 1, 852 35 Bratislava, Slovakia.

[c] Institute of Financial Complex Systems, Masaryk university, Lipová 41a, 60200 Brno, Czech Republic.

[d] Librade LTD, 20-22 Wenlock Road, London, N1 7GU, England.

[*] Corresponding author. E-mail: tomas.vyrost@euba.sk

**Acknowledgement**

We appreciate support received from the Slovak Grant Agency (VEGA No. 1/0257/18). This work was supported by the Slovak Research and Development Agency under Contract No. APVV-14-0357.

## 1. Introduction

Portfolio selection is a key concept of modern finance theory. Since the seminal work of Markowitz (1952), a vast amount of literature has focused on developing new concepts and techniques for more effective portfolio diversification. This includes Black and Litterman's (1991) model, which enables investors' specific views on asset returns to be included in portfolio optimizations; the so-called post-modern portfolio theory (Rom and Ferguson, 1994), which recognizes downside risks other than normal distributions for patterns of returns; the strand of literature that considers higher-order moments of returns in optimization procedures (e.g., Kane, 1982; de Athayde and Flôres, 2004); and improved covariance matrix estimators (Tola et al., 2008; Pantaleo et al., 2011).

In this paper, we explore the potential benefits that information on the interconnectedness of returns offers across five asset classes: stocks, commodities, currencies, bond and money market instruments. Motivated by the recent contribution in the literature (Peralta and Zareii, 2016), our contribution lies in proposing and comparing asset allocation strategies that exploit the topological structure of a network of relationships across assets. Our analysis is conducted over four steps. First, we construct the return series for each asset and calculate correlations among returns. Second, we construct various time-varying correlation networks and quantify the relative importance of assets within the network based on their interconnectedness, i.e., the centrality measure. Third, we construct investment portfolios: either benchmark portfolios based on modern portfolio theory or their alternative versions augmented by information on network topology. Here, we propose a simple constraint in portfolio optimization that requires weights to respect the ordering imposed by centrality measures. Fourth, we focus on various measures of the constructed portfolios to evaluate their performance.

Our findings suggest that our alternative network-based asset allocation strategies improve risk-return profiles relative to the benchmark portfolio. Moreover, the improved risk-return profile is robust with respect to transaction costs, which is especially important from a practical point of view.

Our study is closely related to the work of Peralta and Zareii (2016), who were the first to theoretically prove the negative relationship between the centrality of assets within a financial network and optimal weights under the Markowitz framework. As a result, the centrality measures of constructed networks can be used to facilitate portfolio selection. This has also been demonstrated empirically by means of in-sample and out-of-sample analyses of network-based investment strategies, which may improve portfolio performance. As these authors note, much of the network-related research lacks specific practical applications in terms of portfolio selection and is more descriptive in nature. We fill this gap in the literature, and our results clearly show that the constructed network takes into account complex relationships between assets beyond those measured by correlations. Our main contributions can be summarized in more detail as follows:

i. When transaction costs are ignored, simple extensions of network-based asset allocation strategies generally improve risk-return characteristics.

ii. When the goal is to maximize portfolio returns, improved risk and left-tail risk-adjusted returns are costly, as they are mitigated by increased turnover and transaction costs.
iii. When the goal is to minimize portfolio risks, improved risks and left-tail risk-adjusted returns are likely to be retained even when transaction costs are included, as transaction costs are lower than those of Markowitz's respective risk minimization portfolio.
iv. The network approach appears to improve portfolios irrespective of the employed network model or centrality measure used.
v. The most notable improvements were found with regard to left-tail risk-adjusted returns.
vi. Resolving portfolio model choice uncertainties by using combination portfolios (where 50% of investor wealth is invested in the benchmark portfolio while 50% is invested into a set of network-based asset allocation portfolios) leads to improved risk-return characteristics and is the recommended approach.

## 2. Interconnectedness among various asset classes

As network theory has penetrated the field of finance (e.g., Mantegna, 1999; Mantegna and Stanley, 1999; Onnela et al., 2003), several authors have recommended exploiting the benefits of networks in portfolio selection. Within this network perspective, financial markets or individual assets are perceived as nodes and links among them are usually represented by correlations. As such, correlation networks can be used to establish links among numerous asset classes and to specify which assets are important in terms of interconnectedness (based on some centrality measure). The usefulness of this approach has been supported by Billio et al. (2012) and Diebold and Yilmaz (2014, 2015), who have already suggested that further research on complex relationships in financial markets is headed toward a reliance on network approaches.

As argued by Baitinger and Papenbrock (2016), interconnectedness as an alternative risk concept has thus far attracted very little attention in the field of portfolio management both in theory and practice. They show that interconnectedness risk (i) shows only moderate or no connection to conventional portfolio optimization inputs and that (ii) active investment strategies based on interconnectedness information outperform their conventional counterparts. Kaya (2015) describes how networks among asset classes (based on mutual information distance) can be used to measure and visualize systemic risk, to enhance diversification and to assist with asset pricing. In building diversified portfolios, López de Prado (2016) combines graph theory (hierarchical structure known as a tree) and machine learning techniques to address problems of a quadratic optimization procedure designed for inequality-constrained portfolio optimization problems (known as the Critical Line Algorithm proposed by Markowitz, 1956). Onnela et al. (2003) note that assets of the classic Markowitz portfolio are always located on the outer leaves of the correlation-based minimum spanning tree.

Empirical research has explored potential diversification benefits across different asset classes by studying co-movement between asset returns. For example, the relationship among equities and bonds was studied in an influential paper by Shiller and Beltratti (1992) who found negative co-movement between stocks and bonds; many later studies have produced similar findings (e.g., Andersen et al., 2007; Baele et al., 2010). These negative stock-bond return correlations are mostly described as a "flight-to-safety" phenomenon, or can be explained by the "discount factor", i.e., within the context of present value, increase in interest rates impact both the bond and stock prices, but in the opposite direction. However, later studies (e.g. Andersen et al., 2007) suggest that the negative stock-bonds relation holds only during the contractions in the business cycle, while during the expansion this relationship is positive. In fact, stock-bond correlations exhibit significant time variation (for a persuasive example, see Figure 1 in Baele et al., 2010).

There are several theoretical reasons explaining mutual dependencies among various asset classes. For example, exchange rates and interest rates are closely related via the uncovered interest rate parity (Ciner et al., 2013). Moreover, exchange rates and equities should be closely related, as changes in currency affect a country's trade balance, its output, and thus corporate cash flows. Earlier studies usually reported a positive relation between U.S. dollar revaluations and equity returns, but again, this relationship might be doubted. Ehrmann et al. (2011) found that whereas U.S. equity markets do not respond to exchange rate movements, the euro area markets rise by a substantial amount following an appreciation of the euro. Further, a depreciation of the dollar leads to an increase in U.S. bond yields and a reduction in euro bond yields.

The same ambiguity applies for the oil and equity relationship; an increase in oil prices driven by an increase in demand in the world economy should be associated with positive stock price movements (Park and Ratti, 2008). On the other hand, the results of Cunado and de Gracia (2014) suggest the existence of a negative and significant impact of oil price changes on stock market returns, mostly driven by oil supply shocks. With respect to exchange rates, oil-exporting countries (oil-importing) could experience exchange rate appreciation (depreciation) when oil prices rise (fall) (Krugman, 1980). More recently, Reboredo et al. (2014) showed that in the pre-crisis period oil price changes had a weak and negative effect on exchange rates (and vice versa). Thus, after the recent global financial crisis, there is some evidence of negative interdependence between oil prices and exchange rates. Reboredo (2013) also examined price co-movements between oil and gold. His analysis revealed that gold cannot hedge against oil, but gold can act as a "safe haven" against extreme oil price movements.

The literature on finding the so-called "safe haven" assets, which should provide investors the full benefits of diversification, is quite extensive. The most common candidate to be considered a safe haven is usually gold. Baur and Lucey (2010) studied constant and time-varying relations between U.S., U.K. and German stock and bond returns (MSCI indices) with respect to gold returns. Their results indicate that gold is not a hedge against bonds; however, for stocks, it is a hedge as well as a

safe haven (uncorrelated or negatively correlated asset in extreme market conditions) – but only for a limited time (around 15 trading days). Baumöhl and Lyócsa (2017) showed that the safe haven properties of gold have a changing nature. They examined the quantile dependence among gold and the U.S. stock market sector indices over the period of 1999 – 2016. From a short-term perspective (up to 10 days) after the crisis, gold acted as a safe haven for all sectors except Industrials. From the same short-term perspective but in the period before the financial crisis, they did not find any quantile dependence, and gold could be considered a safe haven for most of the sectors.

Grisse and Nitschka (2015) analyzed the Swiss franc's diversification properties and found that the Swiss franc exhibits safe haven asset characteristics against some currencies but not against other major currencies, such as the U.S. dollar and the yen. Fatum and Yamamoto (2016) investigated currency safe haven during the recent financial crisis and confirmed that during the global crisis, the Japanese yen exhibits the strongest safe haven currency behavior followed by the Swiss franc and then the U.S. dollar. Flavin et al. (2014) assessed a number of safe haven assets from the perspective of an equity investor and provide evidence in favor of choosing either gold or the longer-dated bond as a safe haven asset. Both of these assets delivered risk reduction benefits during the times of stock market decline. In contrast, shorter-dated bonds do not provide such benefits as they are more prone to systemic risk.

Mensi et al. (2013) examine return and volatility links among the S&P500 and commodity price indices for energy, food, gold, and beverages from 2000 to 2011 and find that the gold and oil markets appear to be strongly influenced by U.S. stock market volatility. Nazlioglu et al. (2013) study volatility transmission between oil and selected agricultural commodity prices (wheat, corn, soybeans, and sugar) from 1986 to 2011. Their results from variance causality tests differ depending on the periods examined but reveal significant volatility spillovers from the oil market to the commodity markets (except for sugar) during the post-crisis period. Baruník et al. (2015) analyze volatility spillovers on the oil commodity market over the 1987–2014 period and show that spillovers increase after 2008. However, they also show that relatively balanced and low asymmetries in volatility spillovers correlate well with the ongoing financialization of oil commodities and the advent of heightened oil exploration and production in the U.S. In addition, Baruník et al. (2016) analyze most liquid U.S. stocks in seven sectors and offer ample evidence of the asymmetric connectedness of stocks at the disaggregate level. The asymmetries in spillovers propagate in such a way that although negative spillovers are often of substantial magnitude, they do not strictly dominate positive spillovers. As was the case in the commodities' markets, the overall intra-market connectedness of U.S. stocks is shown to have increased substantially over the recent financial crisis.

The overview is far from exhaustive, but it illustrates a key point. Regardless of asset class pairs, methodology and/or sample period, relationships seem to vary over time. Interconnectedness as an alternative risk concept should thus be incorporated in portfolio optimization.

## 3. Data and methodology

As we allow for diversification across asset classes, we must address the ways in which returns are constructed so that the proposed allocation truly reflects the returns an investor would obtain (this is particularly important when mixing stocks with bonds). We define the rules for portfolio construction that would represent our investment strategies. These strategies include classical approaches to portfolio construction, which may serve as a benchmark but also strategies that are based on the topological properties of underlying time-varying correlation-based networks.

Table 1 Descriptive statistics of weekly asset returns

| | Mean | St.dev. | Skew | Kurt | AC(1) | $AC^2(1)$ | Obs |
|---|---|---|---|---|---|---|---|
| **Stocks** | | | | | | | |
| BVSP | 0.185 | 3.889 | -0.409 | 5.768 | -0.079 | 0.173 | 886 |
| DAX | 0.069 | 3.322 | -0.640 | 7.763 | -0.042 | 0.194 | 886 |
| FTSE100 | 0.004 | 2.491 | -1.064 | 14.242 | -0.085 | 0.090 | 886 |
| KOSPI | 0.144 | 3.610 | -0.503 | 6.764 | -0.081 | 0.314 | 886 |
| MERV | 0.319 | 4.829 | -0.408 | 7.146 | 0.035 | 0.168 | 882 |
| N225 | 0.011 | 3.024 | -1.154 | 11.428 | -0.012 | 0.120 | 886 |
| SMI | 0.012 | 2.662 | -1.107 | 16.647 | -0.161 | 0.278 | 886 |
| SP500 | 0.059 | 2.514 | -0.716 | 9.341 | -0.080 | 0.284 | 886 |
| SSE | 0.109 | 3.472 | -0.084 | 5.072 | 0.068 | 0.139 | 878 |
| TSE | 0.074 | 2.471 | -0.919 | 9.800 | -0.114 | 0.404 | 886 |
| TWII | 0.005 | 3.214 | -0.275 | 6.060 | -0.032 | 0.081 | 873 |
| **Commodities** | | | | | | | |
| Brent | 0.122 | 4.703 | -0.799 | 6.656 | -0.003 | 0.230 | 886 |
| Cocoa | 0.124 | 4.324 | -0.036 | 3.984 | 0.019 | 0.050 | 886 |
| Copper | 0.109 | 3.811 | -0.727 | 8.711 | -0.043 | 0.130 | 886 |
| Cotton | -0.118 | 4.071 | 0.258 | 4.914 | 0.024 | 0.068 | 858 |
| Gold | 0.130 | 2.532 | -0.338 | 6.596 | 0.025 | 0.394 | 886 |
| NatGas | 0.012 | 7.446 | 0.296 | 6.472 | -0.041 | 0.262 | 886 |
| Silver | 0.112 | 4.232 | -0.823 | 6.962 | -0.022 | 0.127 | 886 |
| **Currencies** | | | | | | | |
| AUD_USD | -0.017 | 1.805 | 1.534 | 14.504 | -0.030 | 0.246 | 886 |
| CAD_USD | -0.011 | 1.241 | 0.926 | 9.655 | -0.025 | 0.242 | 886 |
| CHF_USD | -0.039 | 1.625 | -1.126 | 21.705 | -0.031 | 0.029 | 886 |
| CNY_USD | -0.028 | 0.213 | 2.130 | 55.105 | 0.044 | 0.012 | 886 |
| EUR_USD | 0.008 | 1.400 | 0.250 | 3.867 | 0.030 | 0.052 | 886 |
| GBP_USD | 0.009 | 1.287 | 0.846 | 9.439 | -0.030 | 0.213 | 886 |
| JPY_USD | 0.009 | 1.436 | -0.428 | 5.373 | -0.079 | 0.168 | 886 |
| NOK_USD | 0.016 | 1.619 | 0.377 | 3.853 | -0.031 | 0.121 | 886 |
| **Money/bond market** | | | | | | | |
| bAAA | 0.044 | 1.849 | 0.110 | 5.489 | -0.130 | 0.094 | 886 |
| bBBB | 0.029 | 1.536 | -0.399 | 4.701 | -0.070 | 0.213 | 886 |
| bCPF_1M | 0.173 | 4.063 | 2.816 | 40.380 | -0.185 | 0.364 | 884 |
| bCPNF_1M | 0.197 | 2.995 | 3.102 | 25.373 | -0.026 | 0.194 | 885 |
| bEMEA_corp | 0.180 | 0.964 | -3.796 | 53.937 | 0.402 | 0.342 | 886 |
| bEMER_corp | 0.064 | 2.867 | -1.361 | 10.037 | 0.281 | 0.311 | 886 |
| bEMER_corp_high | 0.079 | 1.712 | -1.327 | 10.058 | 0.281 | 0.347 | 886 |
| bEMER_EURO_corp | 0.089 | 2.645 | 1.032 | 26.837 | 0.220 | 0.064 | 886 |
| bEUR_HY | 0.074 | 2.840 | -0.464 | 8.686 | 0.214 | 0.092 | 886 |
| bGER_1Y | 0.220 | 3.045 | 1.230 | 12.860 | 0.018 | 0.198 | 886 |
| bGER_5Y | 0.190 | 3.444 | -0.336 | 6.964 | -0.073 | 0.172 | 886 |
| bGER_20Y | 0.110 | 2.556 | -0.469 | 9.603 | -0.012 | 0.320 | 886 |
| bGER_corp | 0.052 | 2.364 | -0.680 | 14.692 | -0.023 | 0.058 | 886 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| bJPN_1Y | 0.060 | 1.548 | 1.399 | 18.929 | 0.097 | 0.090 | 886 |
| bJPN_5Y | 0.117 | 2.844 | -0.055 | 6.366 | -0.040 | 0.241 | 886 |
| bJPN_20Y | 0.085 | 2.095 | -1.016 | 11.254 | 0.011 | 0.290 | 886 |
| bUS_1Y | 0.167 | 2.975 | 0.716 | 10.880 | 0.032 | 0.233 | 886 |
| bUS_5Y | 0.090 | 3.748 | -0.432 | 4.561 | -0.068 | 0.089 | 886 |
| bUS_20Y | 0.067 | 2.440 | -0.108 | 4.634 | -0.090 | 0.118 | 886 |

*Notes: AC(1) is the value of the first-order autocorrelation coefficient of the original series. $AC^2(1)$ is the value of the first-order autocorrelation coefficient of squares of the original series.*

### 3.1 Data and return series construction

To allow for cross-asset allocation, our sample encompasses stock indices, bonds, commodities and selected foreign exchange rates (see Appendix Table A.1) with a total of $M = 45$ assets (stock market indices, bond and money market instruments, commodities, and foreign exchange rates). The daily return data cover a period from January 1999 to December 2015. Due to non-trading, the number of daily observations for each asset varies from 2614 to 4458. In our analysis, we focus on weekly data, and therefore, given the price $P_{i,w}$ of asset $i$ at the end of the week $w$, the continuous weekly return was calculated as $r_{i,w} = \log(P_{i,w}/P_{i,w-1})$. When the given asset is a bond with a yield $y_{i,w}$, the return is calculated as $r_{i,w} = \log((1+y_{i,w-1})/(1+y_{i,w}))$. In several cases, less than 3 daily observations were available for a given week of a given asset. Such weeks were removed from the given asset time series of weekly returns.

### 3.2 Long-run correlation (variance-covariance matrix)

After creating the return series, we constructed estimation windows for 12 months that were rolled one week ahead, resulting in $w = 1, 2, \ldots, N$ overlapping estimation windows. For each estimation window, we have calculated the long-run correlation coefficient $\rho_{i,j,w}$ for any two return series $r_{i,w}$ and $r_{j,w}$, where $w$ denotes that a given observation belongs to a given estimation window, i.e., that for calculation data from the 52 week prior to $w$ are used. Between two assets $i$ and $j$, the weeks for which returns are recorded may differ, and prior to the calculation of $\rho_{i,j,w}$, series $r_{i,w}$ and $r_{j,w}$ were synchronized via listwise deletion.

To measure the dependence between assets, we decided to use the long-run correlation coefficient, which is based on the estimator of the heteroskedasticity and autocorrelation consistent variance-covariance matrix introduced by Andrews (1991). This choice was guided by the fact that return series are subject to mild levels of autocorrelation and to heteroskedasticity (see Table 1). We also use the long-run estimator, as it is much simpler to use in practice.[1]

For a given sample size $T$, Andrews' (1991) estimate takes the following form:

$$\hat{\mathbf{\Omega}}_{\mathrm{T}} = \begin{bmatrix} \hat{\omega}_{i,i} & \hat{\omega}_{i,j} \\ \hat{\omega}_{j,i} & \hat{\omega}_{j,j} \end{bmatrix} = \sum_{m=-T+1}^{T-1} k\left(\frac{m}{B}\right)\hat{\mathbf{\Gamma}}_T(m) \tag{1}$$

where:

[1] We also experimented with partial correlations as a measure of dependence, but the overall results remained practically the same. These results are available upon request.

$$\mathbf{\Gamma}_{\mathrm{T}}(m)=\begin{cases} T^{-1}\sum_{t=m+1}^{T}\left[\mathbf{Z}_t\mathbf{Z}_{t-m}\right], & m\geq 0 \\ T^{-1}\sum_{t=-m+1}^{\mathrm{T}}\left[\mathbf{Z}_{t+m}\mathbf{Z}_t\right], & m<0 \end{cases} \tag{2}$$

and where $t = 1, 2, \ldots, T$, $Z_t = [r_{i,t}, r_{j,t}]^T$, and $k(.)$ is the quadratic spectral kernel weighting function that together with bandwidth parameter $B$ weights lagged variances and co-variances. In our empirical work, we made an arbitrary choice for the bandwidth parameter to be of size 3, which corresponds to 3 weeks that attain the largest weight. The quadratic spectral kernel function is defined as:

$$k\left(x=\frac{m}{B}\right)=\frac{25}{12\pi^2x^2}\left(\frac{\sin\left(6\pi x/5\right)}{6\pi x/5}-\cos\left(6\pi x/5\right)\right) \tag{3}$$

Finally, the long-run correlation is estimated as:

$$\hat{\rho}_{i,j}\equiv\frac{\hat{\omega}_{i,j}}{\sqrt{\hat{\omega}_{i,i}\hat{\omega}_{j,j}}} \tag{4}$$

A significance test on the long-run correlation coefficient can be performed using Panopoulou et al.'s (2010) method, where the test statistic for $H_0$: $\rho_{i,j} = 0$, is defined as:

$$\hat{\rho}_{i,j}\sqrt{\frac{T}{B}} \tag{5}$$

and where under the null hypothesis, it follows a standard normal distribution. As the test is performed between all possible pairs of 45 assets, we use a rather strict significance level of $0.01/(M(M–1)/2)$ corresponding to the Bonferroni adjustment.

### 3.3 Networks

The calculated correlation coefficients have been used as a basis for the construction of time-varying networks, which are in turn used in portfolio optimization. More specifically, a network at time $w = 1, 2, \ldots, N$ is a graph $G_w(V,E_w)$ defined by a set of vertices $V = \{1, 2, \ldots, M\}$ that corresponds to individual assets and a set of edges $E_w \subseteq V \times V$. Each edge may be assigned a non-negative weight, expressing the "distance" between the vertices. Distances are calculated from long-run correlation coefficients $\rho$ as $d=\sqrt{2(1-\rho)}$. The larger the distance the less interconnected asset returns are. Based on which edges are retained, in the next subsections, we discuss several approaches to the construction of set $E_w$.

#### 3.3.1 Complete graph

As correlations between the returns of any two assets can be calculated at any time, the resulting structure forms a so-called complete graph in which any two vertices are connected by an edge. Although the edge weights (given by correlations) can vary, the resulting structure is rather uninteresting, at least from a topological point of view, as every asset is linked to all others regardless

of the strength of the relationship. A complete graph does not limit the portfolio optimizer in any way with respect to constraints placed on the topology of a network.

In financial network studies, one typically does not analyze complete graphs, but rather a suitable subgraph: a network of the same number of vertices that retains only certain edges that are considered relevant.

In our analysis, we use three different types of subgraphs, namely, a minimum spanning tree, a planar maximally filtered graph, and a threshold significance graph, which are described further in the following subchapters. To obtain a better perspective, we also plot these subgraphs in Figures 1–3 using our full sample. Based on eigenvalue centrality, the London stock market appears to be the one with the most connections in all three subgraphs. Although exclusions of some edges according to different network creation rules do not have straightforward economic meanings, in our figures, it is quite clear that vertices are clustered into asset class groups. This applies to all three of the subgraphs considered.

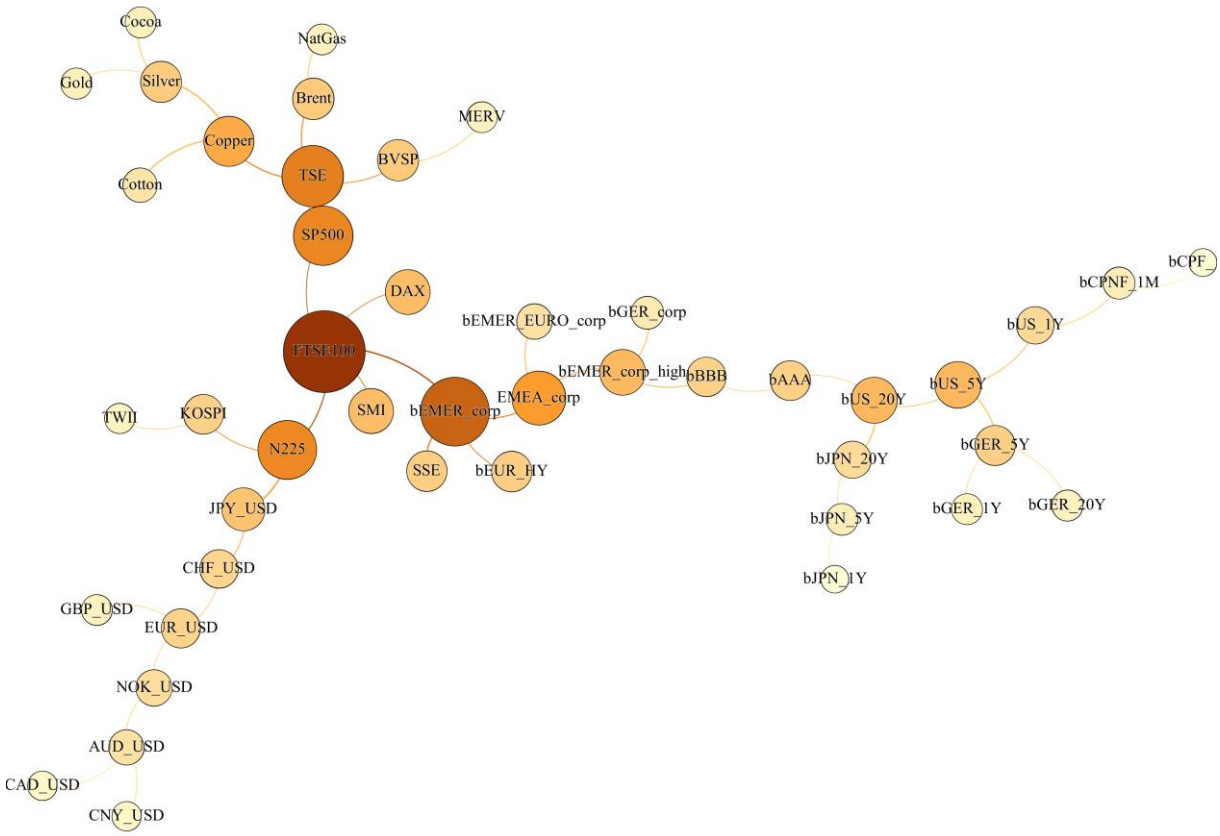


Figure 1 Minimum spanning tree of individual asset return correlations for the full sample

*Notes: Vertex sizes and colors are based on eigenvalue centrality, i.e., the higher the size or the darker the color of the vertex, the higher is the centrality of the given vertex (asset). Edges between vertices represent relationships retained in the minimum spanning tree. Assets that have many edges and are centralized in the graph are considered as more interconnected – an undesirable property for asset allocation problem. For example, FTSE 100 is highly interconnected not only among stock market indices but also within the whole network. On the other hand, gold or the CNY/USD exchange rate are less interconnected with the rest of the assets.*

### 3.3.2 Minimum spanning tree

A widely used and well-established subgraph is the so-called *minimum spanning tree* (MST), which was introduced to financial contexts by Mantegna (1999). A spanning tree is an acyclic connected subgraph containing all vertices (a graph with no circles) with a path connecting any two vertices. Requirement of a minimal such spanning tree refers to the values of edge weights. To satisfy the conditions of edge weight non-negativity, which allows for their interpretation as distances, Mantegna (1999) proposed a nonlinear decreasing transformation of correlations to be used for weights $\left(d_{i,j} = \sqrt{2(1-\rho_{i,j})}\right)$. As the transform is decreasing, higher correlations translate into smaller distances. MST is thus a spanning tree with a minimum sum of weights of retained edges. This supports the notion of keeping only the most important edges in a graph intact. In a network of *M* vertices, an MST retains precisely *M*–1 edges. The MST is extracted from the complete graph using Kruskal's (1956) algorithm.

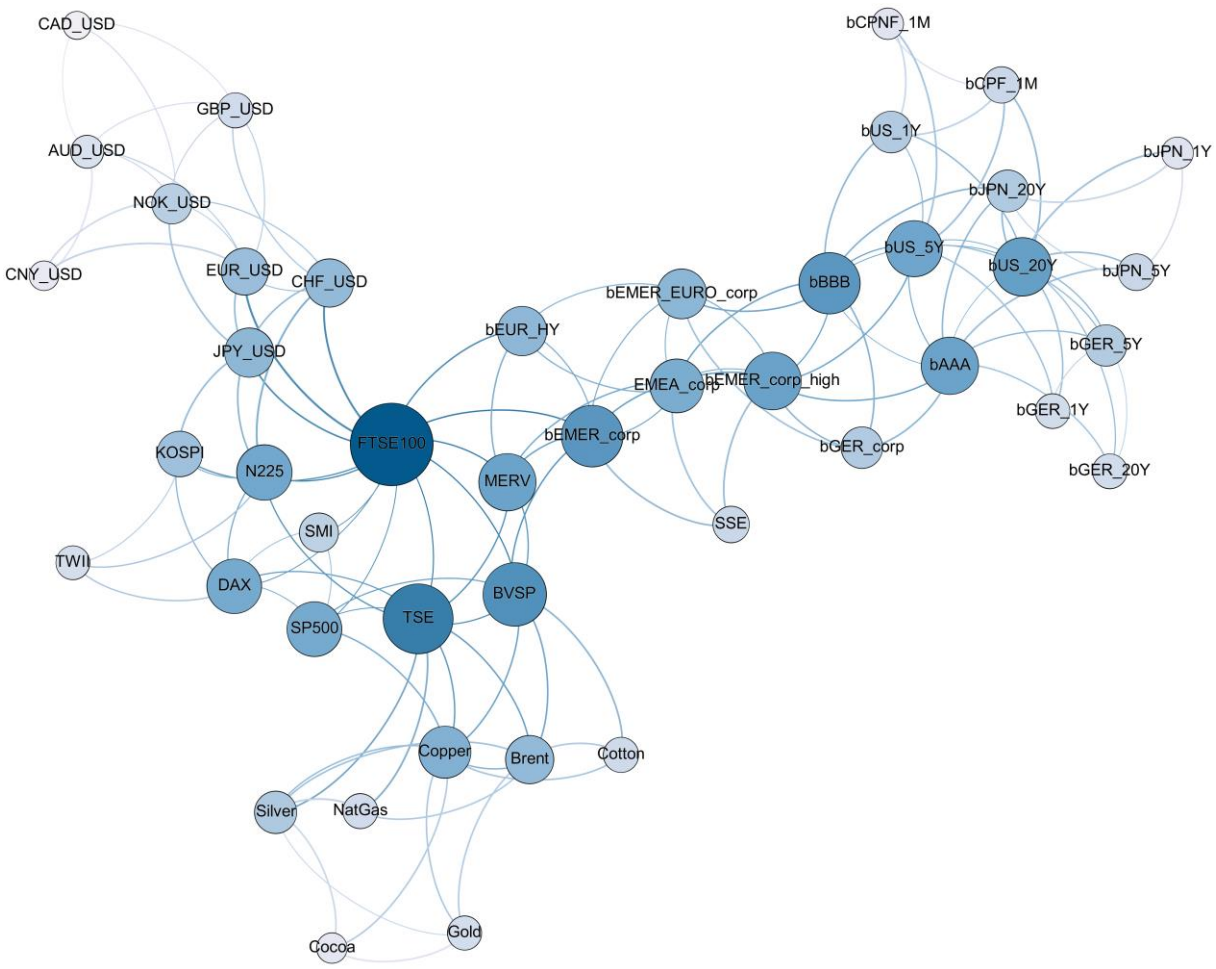


Figure 2 Planar maximally filtered graph of individual asset return correlations for the full sample

*Notes: Vertex sizes and colors are based on eigenvalue centrality, i.e., the higher the size or the darker the color of the vertex, the higher is the centrality of the given vertex (asset). Edges between vertices represent relationships retained in the planar maximally filtered graph. As before, FTSE 100 is interconnected among stock market indices. Among bonds, it is the 20-year U.S. Treasury bond, while among commodities, it is Copper.*

### 3.3.3 Planar maximally filtered graph

Although the MST presents a frequently used subgraph, its simplicity may prove limiting. As a complete (unoriented) graph on $M$ vertices has $M(M-1)/2$ edges, its reduction to an MST might translate into a loss of some important network features. To allow for a richer set of network structures, Tumminello et al. (2005) proposed using a *planar maximally filtered graph* (PMFG) that replaces the constraints of a spanning tree with constraints of forming a planar graph. These subgraphs may include cycles or complete subgraphs of up to 4 vertices and they are thus more suitable for describing highly interconnected networks. A PMFG on $M$ vertices has precisely $3M-6$ edges, which is roughly three times as much as a minimum spanning tree. In addition, an MST is a subgraph of a PMFG; therefore, a PMFG includes all of the edges of an MST and may be thought of as its extension.

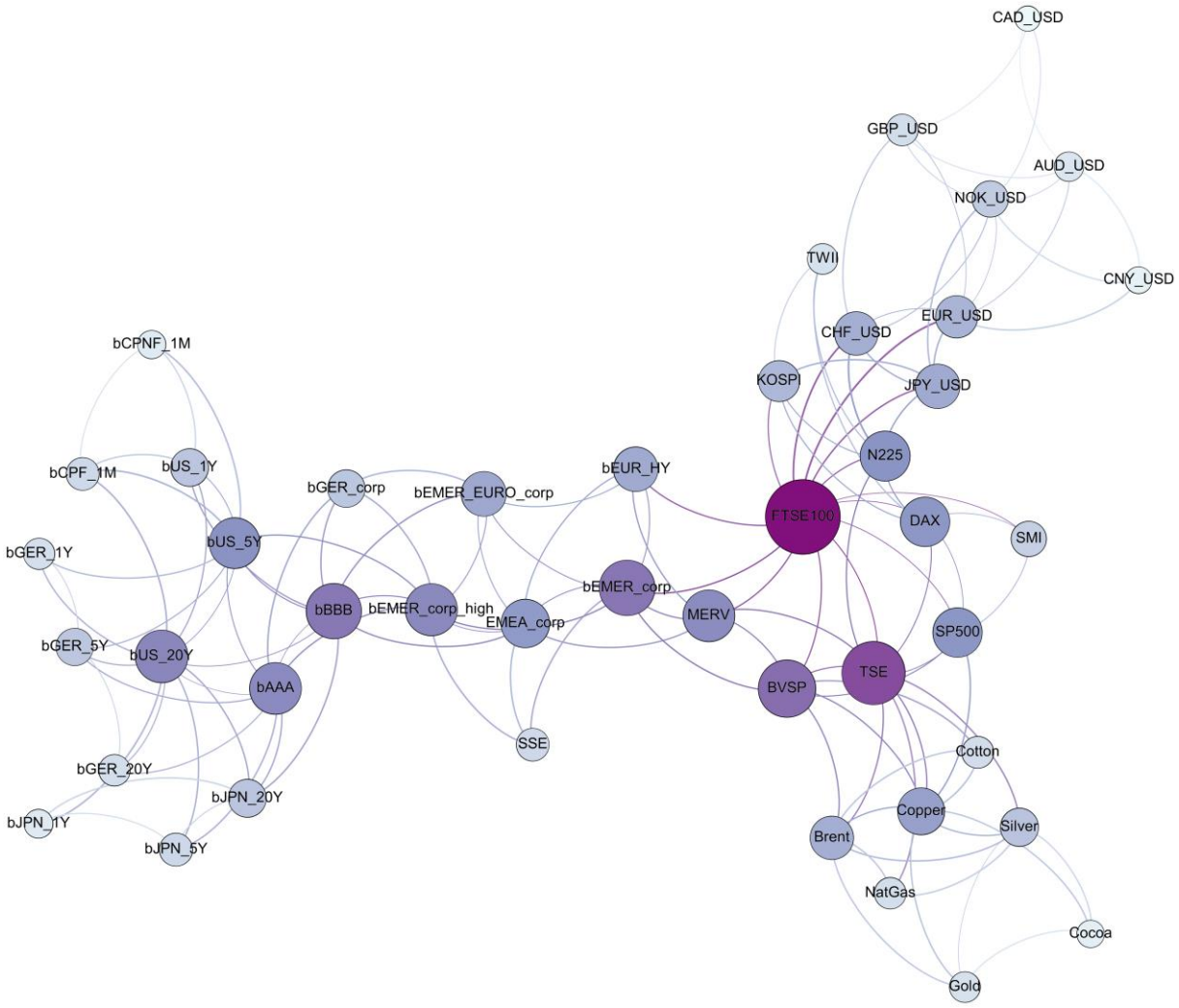


Figure 3 Threshold significance graph of individual asset return correlations for the full sample

*Notes: Vertex sizes and colors are based on eigenvalue centrality, i.e. the higher the size or the darker the color of the vertex, the higher is the centrality of the given vertex (asset). Edges between vertices represent relationships retained in the threshold significance graph. Additionally, the threshold significance graph revealed the central role of the FTSE 100 among all assets.*

### 3.3.4 Threshold significance graph

Rather than creating topological constraints on the interconnectedness of assets, Tse et al. (2010) considered networks wherein only correlations above a threshold value were used, as topological constraints in MST and PMFG have unclear statistical and economic meaning (Výrost et al., 2015). We therefore also considered threshold networks where only edges that were statistically significant remained in the network. More specifically, when the estimate of $\rho_{i,j}$ was insignificant from zero, the estimate of $\rho_{i,j}$ was set to 0.

## 3.4 Centrality measures

To describe the topology of the financial networks, we use several measures of the relative importance of assets. We focus mainly on centrality measures, namely, betweenness and eigenvalue centrality. Each centrality represents a different measure, thus resulting in a numerical nodal attribute describing the importance of a node relative to others. The simplest centrality measure, which we do not use to construct our portfolios, is the degree centrality, which assigns each vertex to the number of incident edges; thus, the higher the degree of centrality, the more interconnection a vertex has with remaining vertices in a network.

*Betweenness* represents a different centrality measure that counts the number of times a vertex lies on the shortest path between other vertices in the network. The intuition behind betweenness is as follows. Assume that we wish to calculate the measure for a vertex $v \in V$. For any two distinct vertices other than $v$, say $s$, $t \in V$, the shortest path may be deemed the most direct path information (or return shocks) may take to spill over between $s$ and $t$. As the distance between vertices is given by edge weights, which are a decreasing function of their mutual correlations, shortest paths are routes along the highest correlations to reach $s$ from $t$ (and vice versa). Depending on the network structure, there may be more than one shortest path between any two vertices. For example, in an MST, there is always only one shortest path; in a PMFG, there will always be at least one such path, and in a complete graph, shortest paths will always reduce to a single edge, as all vertices are adjacent. To calculate betweenness for $v$, one counts the fraction of times a shortest path between $s$ and $t$ contains $v$. More formally, let the number of shortest paths between $s$ and $t$ be $n_{s,v,t} \in \mathbb{N}$ and let $n_{s,t} \in \mathbb{N}$ be the total number of shortest paths between $s$ and $t$. The betweenness for $v$ is then calculated as:

$$\sum_{\substack{s \neq v \neq t \\ s,v,t \in V}} \frac{n_{s,v,t}}{n_{s,t}} \tag{6}$$

In cases of betweenness, vertices with high centrality present important nodes in the sense that they mediate the interconnection between other vertices and act as spillover hubs.

Another centrality measure we use in this study is *eigenvalue centrality*. Here, the relative importance of a vertex not only depends on the number of connections a vertex has, as their quality

also becomes relevant. As is the case in social networks, some vertices may be more significant than others, and in turn connections to these vertices are more important than connections to peripheral vertices. Being connected by a few links to a small number of important vertices may be more relevant than forming a higher number of connections with inconsequential vertices. To capture this idea more formally, let $\mathbf{A}_t$ be the adjacency matrix of $G_t$, that is, an $M \times M$ matrix, where $a_{s,t} = 1$ if there is an edge between vertices $s$ and $t$ and where $a_{s,t} = 0$ otherwise. The centrality measure for all vertices is obtained by finding the greatest eigenvalue $\lambda \in \mathbb{R}$ such that for a vector $\mathbf{x} \in \mathbb{R}^M$:

$$\lambda \mathbf{x} = \mathbf{A}\mathbf{x} \tag{7}$$

Eigenvalue centrality scores may be extracted as elements of the eigenvector $\mathbf{x}$. Note that for (strongly) connected graphs, such as those referenced in this paper, the eigenvector solution is unique and positive as a consequence of the Perron-Frobenius theorem.

As a third possible measure of the relative importance of an asset in a network, we use the *expected force*. As Lawyer (2015) argues, classical centrality measures are usually well suited for identifying the most important vertices in a network, but their values are not very meaningful for other, less important nodes. In our case, when determining portfolio weights depending on centrality measures, the importance of all vertices is critical. The expected force is a measure used to describe the spreading power of nodes from an epidemiological perspective. In a financial context, this property becomes relevant in describing a situation wherein development on some assets spills over to other assets or asset classes (as is the case for contagion). From the expected force, it is possible to capture two distinct aspects that define the ability of a vertex to spread information through a network. For vertices with high spreading power, their force comes from their own interconnectedness (or vertex degree). However, vertices with less spreading power may also exhibit high levels of expected force when their direct neighbors have high vertex degrees and are thus able to spread information through the network structure more quickly.

In general, we expect the centrality measures to provide additional information to the problem of portfolio optimization. The main reason for this belief is the fact that in normal econometric or statistical setting, most relationships are captured as bivariate, such as in the case of correlation matrices. Network analysis adds a unique perspective on interrelationships: most centrality measures are based on paths, rather than bivariate relationships. Shortest paths and their lengths present a form of measure of interconnectedness that cannot be replicated by current econometric tools, which makes them a potentially useful alternative in problems exploiting dependence structures, as is the case in portfolio optimization.

### 3.5 Portfolio strategies

Specifications of the optimization problems described in this section are used to calculate optimum portfolio weights, thus determining the share of an investor's capital invested into respective assets.

When trying to incorporate topological information from financial networks into an optimization problem, there are two alternatives: topological information can either become part of the objective function and thus become the determinant and measure of suitability of a proposed portfolio regarding what an investor deems desirable, or it is possible to include such information as additional constraints that a portfolio must satisfy. In this paper, we pursue the second approach only.

To explain the consequences of this decision, we may consider the interpretation of both approaches. Changing the objective function in a portfolio optimization routine might be viewed as changing the property the investor desires – may it be maximum return, minimal volatility or some other measure. Changing the constraints on the other hand only limits the feasible set of portfolios that are acceptable for an investor.

The classical mean-variance theory has been derived from maximization of investor's utility approximated by a Taylor-series expansion, which leads to the well-known result of utility increasing with the expected return and decreasing with volatility of portfolio returns. In our approach, we follow these classical results and do not make modifications to the objective function, which we consider to be valid.

As for the constraints, several legitimate reasons may exist that motivate a change in the feasible set of portfolios. As a simple example, an investor might wish to control the number of assets held – either to achieve diversification, or to limit the transaction costs. A different constraint may be added that would require the optimal portfolio to exhibit an expected return above (or volatility below) a selected threshold. None of these requirements change the objective function, their purpose is only to refine the set of portfolios under consideration.

In our approach, we set additional constraints, all of which are based on a variation of network centrality. In the classical mean-variance framework, increasing the weight of assets with highly positively correlated returns is undesirable, as the increase in interdependence leads to increased risk due to weak diversification (the diversification effect in case of any two assets is inversely related to correlation). An asset with high centrality may be similarly undesirable – a vertex that is very strongly connected to others may be viewed as risky, as any negative market movement is likely to influence not just the asset, but also its neighbors. By adding constraints for network centrality, we try to control for such risk – by avoiding high centrality, which carries more information than simple bivariate correlation (it is affected by relationships to potentially all other vertices), there is hope for achieving better portfolio performance.

This argument also answers a reasonable question, why would one expect the inclusion of constraints to lead to any benefits over an unconstrained problem. If the problem was deterministic, then the solution to the unconstrained case would never be worse than the constrained one. However, the returns are stochastic processes, and therefore, in an out-of-sample framework, the usefulness of the constrained solutions needs to be tested empirically.

The beauty of Markowitz mean-variance framework lays in its generality, as it follows from very simple and reasonable assumptions. As a consequence, it does not address all aspects of risk that may be relevant to an investor, such as downside risk, skewness or fat tails of return distributions. Our expectations of future returns and volatility might be different from simple historical averages – if, for example, we use a forecasting model for mean or volatility with superior performance, this may influence our choice of optimal weights. Similarly, in this paper, we explore the question of whether utilizing centrality measures of interconnectedness adds meaningful information and leads to better portfolio performance in an out-of-sample framework.

For robustness, we use four different network subgraphs (complete, MST, PMFG and threshold graphs) together with three centrality measures (betweenness, eigenvalue centrality and expected force) to augment the return-maximization and risk-minimization strategies.

### 3.5.1 Return maximization strategies

The benchmark strategies we use are widely recognized in portfolio theory and include the maximization of expected returns and a variance minimizing objective. As the maximization of expected returns would by itself almost always allocate all funds to a single asset (with the exception of possible equal maximal expected returns for some assets), this would hardly be considered a useful portfolio strategy, at least from the point of view of risk diversification. We therefore enforce a quadratic constraint on the expected variance of the optimal portfolio, which should not be higher than the average volatility observed across all assets. The optimization problem can be more formally written as:

$$\begin{aligned} &\arg\max_{\boldsymbol{\alpha}\in R^M} \boldsymbol{\alpha}^T E(\mathbf{r}) \\ &\boldsymbol{\alpha}^T D(\mathbf{r})\boldsymbol{\alpha} \le \frac{tr(D(\mathbf{r}))}{M} \\ &\boldsymbol{\alpha}^T \mathbf{1} = 1 \\ &\alpha_i \ge 0, \quad i = 1, 2, ..., M \end{aligned} \tag{8}$$

where $\boldsymbol{\alpha}$ is a vector of portfolio weights of length $M$, $E(\mathbf{r})$ is a vector of expected returns, $D(\mathbf{r})$ is the $M \times M$ covariance matrix of returns, $tr(D(\mathbf{r}))$ is the trace of the covariance matrix (sum of its diagonal elements), and $\mathbf{1}$ is a vector of length $M$ with all elements equal to 1. In practical applications, $E(\mathbf{r})$ is replaced by historical average returns and $D(\mathbf{r})$ by their variance.

### 3.5.2 Risk minimization strategies

A second classical strategy of portfolio construction involves risk minimization. As the main benefit of constructing portfolios is diversification, minimizing the expected variance or the volatility of a portfolio may be desirable. However, such strategies lead to the generation of almost non-

competitive returns, and we therefore decided to impose one additional constraint on the expected return that should not be lower than the average return observed across all assets:

$$\begin{aligned} &\arg\min_{\boldsymbol{\alpha}\in R^M} \boldsymbol{\alpha}^T D(\mathbf{r})\boldsymbol{\alpha} \\ &\boldsymbol{\alpha}^T E(\mathbf{r}) \geq \frac{\mathbf{1}^T E(\mathbf{r})}{M} \\ &\boldsymbol{\alpha}^T \mathbf{1} = 1 \\ &\alpha_i \geq 0, \quad i = 1, 2, ..., M \end{aligned} \tag{9}$$

### 3.5.3 Alternative network-based asset allocation strategies

Network information can be used to limit a portfolio's exposure to a certain asset by not allowing its weight to become too large relative to others. This can be achieved by adding a constraint that requires weights to respect the ordering imposed by centrality measures. We decided to add the following constraint to the set of constraints defined above (see Sections 3.5.1 and 3.5.2):

$$b_i \geq b_j \quad \Rightarrow \quad \alpha_i \leq \alpha_j, \qquad i, j = 1, 2, ..., M \tag{10}$$

Here, $b_i$ is the betweenness of the $i$-th asset in a given network. The simple constraint defined above was used for each of the centrality measures (the betweenness, eigenvalue, or expected force) and for each of the networks employed (complete, MST, PMFG, and threshold networks). We in turn generated twelve alternative network-based asset allocation strategies for which the objective was to maximize returns (see Section 3.5.1), and we created twelve alternative network-based asset allocation strategies for minimizing risk (see Section 3.5.2). However, when a complete network is used, the resulting centrality measures are the same for betweenness and expected force values. Therefore, we only retained the latter strategy.

### 3.5.4 Combination strategies

Since the early works of Bates and Granger (1969), it has been argued that forecasts can be improved by combining several unbiased but not highly correlated forecasts into one forecast. We employ the same principle by always combining the benchmark with several alternative network-based asset allocation strategies. This way, we are able to observe which portfolio performance characteristics of the benchmark portfolio are being improved upon when combined with network-based asset allocation strategies.

For example, in Table 3, we have a strategy denoted as "Betweenness + B". For this strategy, 50% of investor wealth is allocated to the benchmark strategy (in Table 3, it is the return maximization strategy defined in Section 3.5.1), while the remaining 50% is spread equally across all strategies, which use betweenness centrality measures in the constraint (see Section 3.5.3). Our choice of equal weights is motivated by strand of literature following the work of deMiguel et al. (2007), which

documents the difficulties of various approaches to beat the naïve "equal-weights" diversification strategy.

### 3.6 Out-of-sample portfolio performance evaluation

Out-of-sample evaluation is performed within a rolling estimation framework, mimicking an investor with one-week investment horizon. We use the first 52 weeks (approximately 1 year) to create networks (i.e., to estimate long-run correlations, expected historical returns) and subsequently to estimate 'optimal weights' (Eq. (8) and Eq. (9)) of each asset. We use optimal weights for asset allocation purposes with an investment horizon of one week. We calculate and store the realized returns for the 53rd week for each of the portfolios. Next, we roll the estimation window one-week ahead, i.e., we use data from the 2nd to 53rd weeks to estimate new 'optimal weights' that are subsequently used for asset allocation purposes of the 54th week. The procedure is repeated until we reach the end of our sample.

We report descriptive statistics for each of the portfolios and perform model confidence set procedures to compare each portfolio with a given benchmark portfolio in terms of mean returns and different Sharpe ratios.

#### 3.6.1 Portfolio characteristics

Each portfolio is evaluated based on several basic descriptive statistics: the minimum (Min), the maximum (Max), the average portfolio return (Mean), the 1st (Q1) and 3rd (Q3) quartiles of portfolio returns and the standard deviation of portfolio returns (SD). The descriptive statistics are complemented with a set of portfolio-specific measures: the average drawdown (DD), the expected shortfall (ES), the Burke ratio (BR), and the Sharpe ratio (SR). As our sample covers a wide range of international stocks, in the main body of the text we evaluate performance of portfolios assuming a zero risk-free rate, as it is unclear which rate to actually use in such heterogeneous and international sample. In the Appendix (Tables A.2 and A.3), we report results where portfolio performance is evaluated using excess return calculated as the difference of the realized return over the 3-month Treasury bill rate.

If $P_{i,w}$ is the value of the $i$-th portfolio at time $w$, the drawdown is defined as:

$$D_{i,w} = \frac{P_{i,w}}{\max\limits_{t=1,\ldots,w} P_{i,t}} \tag{11}$$

The average drawdown $DD_i$ is the average of $D_{i,w}$ over $w = 1, 2, \ldots, N$. The expected shortfall of the $i$-th portfolio $ES_i^{\alpha}$ is defined as:

$$ES_i^{\alpha} = E\left[r_i \middle| r_i < -VaR^{\alpha}\right] \tag{12}$$

Here, $\alpha$ denotes the left-tail quantile (usually 10%, 5%, and 1%). The Burke ratio, $BR_i$, is defined as the average portfolio return divided by the sum of squared drawdowns:

$$BR_i = \frac{\sum_{w=1}^{N} r_{i,w}}{\sqrt{\sum_{w=1}^{N} D_{i,w}^2}} \tag{13}$$

Comparing two portfolios via the Burke ratio allows us to assess whether the higher drawdown associated with one portfolio is adequately compensated by its average return. In a similar fashion, we employ the widely-used Sharpe ratio and its extensions. The standard Sharpe ratio with a risk-free rate of 0% is defined as:

$$SR_i = \frac{\sum_{w=1}^{N} r_{i,w}}{SD_i} \tag{14}$$

The $SD_i$ is the standard deviation of portfolio returns, and the ratio is used to assess whether the average portfolio return compensates for the variability in portfolio returns. However, it is well known that not all variability is bad for the investor. When an investor is in the long position, variability associated with market declines is to be considered riskier as variability associated with the market increases. We therefore use modified versions of the Sharpe ratio where the denominator is substituted with $ES_i^{\alpha}$. In turn, we assess whether the investor is compensated for left-tail risks.

### 3.6.2 Model confidence set

We use Hansen et al.'s (2011) model confidence set to test whether from a given set of models $S$ with a dimension of $s$, there is a superior subset $S^*$ with a dimension of $s^* \leq s$. In our empirical application, we are interested in comparing portfolio returns and Sharpe ratios. The test is illustrated on an example of using returns as a performance measure. Given that the benchmark return is a risk-free rate $r^f_w$, let $d_{i,j,w}$ denote the differential between returns of portfolio $i$ and $j$:

$$d_{i,j,w} = l_{i,w} - l_{j,w} = \left(r_{i,w} - r_w^f\right) - \left(r_{j,w} - r_w^f\right) \tag{15}$$

where $r$ denotes the given portfolio return, $i, j = 1, 2, \ldots, s$, and $w = 1, 2, \ldots, N$. In the text, we set $r^f_w$ to 0 for all $w$ values (see Section 3.6.1 for a discussion and Appendix Tables A.2 and A.3 for results using the 3-month Treasury bill). Formally, the multiple hypothesis test can be formulated as $H_0$: $E[d_{i,j}] = 0$ for all $i, j$. We use the following test statistics from Hansen et al. (2011):

$$T_S = \max_{i,j \in S} \left|t_{i,j}\right| \tag{16}$$

where $t_{i,j}$ is defined as:

$$t_{i,j} = \frac{\bar{d}_{i,j}}{\sqrt{\mathrm{var}^*\left(\bar{d}_{i,j,w}\right)}} = \frac{\sum_{w=1}^{N} d_{i,j,w}}{N\sqrt{\mathrm{var}^*\left(\bar{d}_{i,j,w}\right)}} \tag{17}$$

and where the denominator is the bootstrap estimate of the variance of the average loss differential between portfolios *i* and *j*. We follow Hansen et al. (2011) and use the block-bootstrap with 5,000 replications. The distribution of the test statistics under the null hypothesis is also bootstrapped (see Hansen et al., 2011). The procedure applied to Sharpe ratios is similar: all returns are divided by the selected risk measure (either the standard deviation or the absolute value of a given expected shortfall in the case of modified Sharpe ratios).

### 3.6.3 Transaction costs

To account for possible transaction cost effects on overall portfolio performance, we follow Peralta and Zareei (2016) and calculate the break-even transaction cost (*BETC*) of each strategy. This approach assumes a fixed cost for each transaction but does not require any assumptions on transaction fees, which would become nuisance parameters in the calculation. Instead, the *BETC* calculates how high costs would have to be to eliminate the return generated by a portfolio strategy. Thus, different strategies may be directly compared without the need to establish specific transaction fees.

The break-even transaction cost is dependent on portfolio returns but also on the turnover. Strategies that are less prone to frequent changes in asset allocations should exhibit lower costs and thus should potentially have a higher *BETC*. We calculate the turnover for each portfolio strategy *i* as:

$$O_i = \frac{1}{M}\sum_{w=1}^{N-1}\sum_{j=1}^{M}\left|\alpha_{i,j,w+1} - \frac{\alpha_{i,j,w}(1+r_{i,j,w+1})}{\sum_{k=1}^{M}\alpha_{i,k,w}(1+r_{i,k,w+1})}\right| \tag{18}$$

Thus, for each time period, we calculate the difference in asset allocations between optimal weights for the next period $\alpha_{i,j,w+1}$ (with the corresponding next period return $r_{i,j,w+1}$) for portfolio *i* and asset *j* and the asset allocation of the previous period, including the change in the relative asset composition resulting from gains or losses between initial investments and the time at which the portfolio is being rebalanced.

The turnover is used in the calculation of $BETC_i$ for each portfolio strategy *i* as the solution of the following equation:

$$\sum_{w=1}^{N}\left[\left(1+r_{i,w}\right)\left(1 - BETC_i\sum_{j=1}^{M}\left|\alpha_{i,j,w+1} - \frac{\alpha_{i,j,w}(1+r_{i,j,w+1})}{\sum_{k=1}^{M}\alpha_{i,k,w}(1+r_{i,k,w+1})}\right|\right) - 1\right] = 0 \tag{19}$$

Here, $r_{i,w}$ is the return of the portfolio strategy *i* at time *w*.

## 4. Results

In this section, we compare alternative network-based asset allocation strategies with respective benchmarks. Rather than going into fine detail, we focus on general differences between individual (Section 3.5.3) and combination strategies (Section 3.5.4) based on respective benchmark strategies (Sections 3.5.1 and 3.5.2).

### 4.1 Evaluation of return maximization strategies

Although the benchmark strategy (V00) generated the highest average portfolio return, except in one case, the highest average portfolio return was not found to be statistically significant (Hansen et al.'s 2011 test) from average returns of individual and combined portfolio strategies. Furthermore, two strategies led to similar levels of average returns (V10 and V16).

The benchmark strategy also generated the highest standard deviation (*SD*), as all alternative network-based asset allocation strategies vary less in terms of portfolio returns. Interestingly, when reviewing individual strategies (V01-V11), *SD* was found to be on average 37.7% smaller than the *SD* of the benchmark strategy. However, the returns were on average 54.6% smaller as well, suggesting that individual network-based strategies have not improved the overall risk-return profile of the benchmark portfolio. This is confirmed by the Sharpe ratios (*SR*), which are in most cases smaller than those reported for the benchmark portfolio.

On the other hand, a review of the combination strategies (V12-V19) shows that the *SD* declined by approximately 37.2% on average, while the return only declined by 27.2%. Therefore, the Sharpe ratios of combination strategies are now larger than the benchmark and in one case (V16) also statistically significant.

These results suggest that network-based asset allocation strategies have the potential to improve the risk-return profile of the benchmark portfolio. This is further supported by additional risk-based measures, as the average drawdown (*DD*) tends to be smaller while the Burke ratio (*BR*) is always larger for combination strategies.

Regarding left-tail risks, expected shortfalls in the combination strategies are always larger (i.e. less negative conditional returns) for all three quantiles (*ES*10, *ES*5 and *ES*1). More importantly, the excess return of the benchmark portfolio does not seem to compensate for the excess left-tail risk, as the modified Sharpe ratios (*SR*10, *SR*5 and *SR*1) for combination strategies tend to get improved.

For illustrative purposes, Figure 4 plots the cumulative returns and drawdown of the benchmark strategy and presents the two individual strategies that performed the best according to the *SR*, *SR10*, and *SR5* portfolio performance measures. The figure clearly shows that the benchmark strategy led to the highest cumulative returns most of the time but also suffered from excessive drawdown periods. The cumulative returns of network-based asset allocation strategies are less volatile and considering

the portfolio performance characteristics described above, it seems that these strategies might be valuable for investors living in mean-variance utility framework.

In Figure 5, asset allocation is visualized for the same set of strategies where we can observe the portion of investor's wealth allocated to specific asset classes (stocks, commodities, currencies, and bonds). Clearly, regardless of the strategy used, stocks seem to dominate dynamics of asset allocation over short periods where either all or no investor wealth is allocated to stocks (see spikes in the blue polygon in Figure 5).

These findings led us to evaluate strategies based on the turnover and BETC. Here, it is evident that not only is the turnover much larger for individual network-based strategies, but the BETC is also smaller, which suggests that although individual strategies led to improved risk-return portfolio characteristics, when transaction costs are taken into account, such benefits may disappear. Turnover and transaction costs of combination strategies are better than those of individual strategies but are still worse than those of the benchmark. We therefore conclude that under the return maximization portfolio approach, the improved risk and left-tail risk-adjusted returns of network-based asset allocation strategies are sacrificed by increased transaction costs (Table 2).

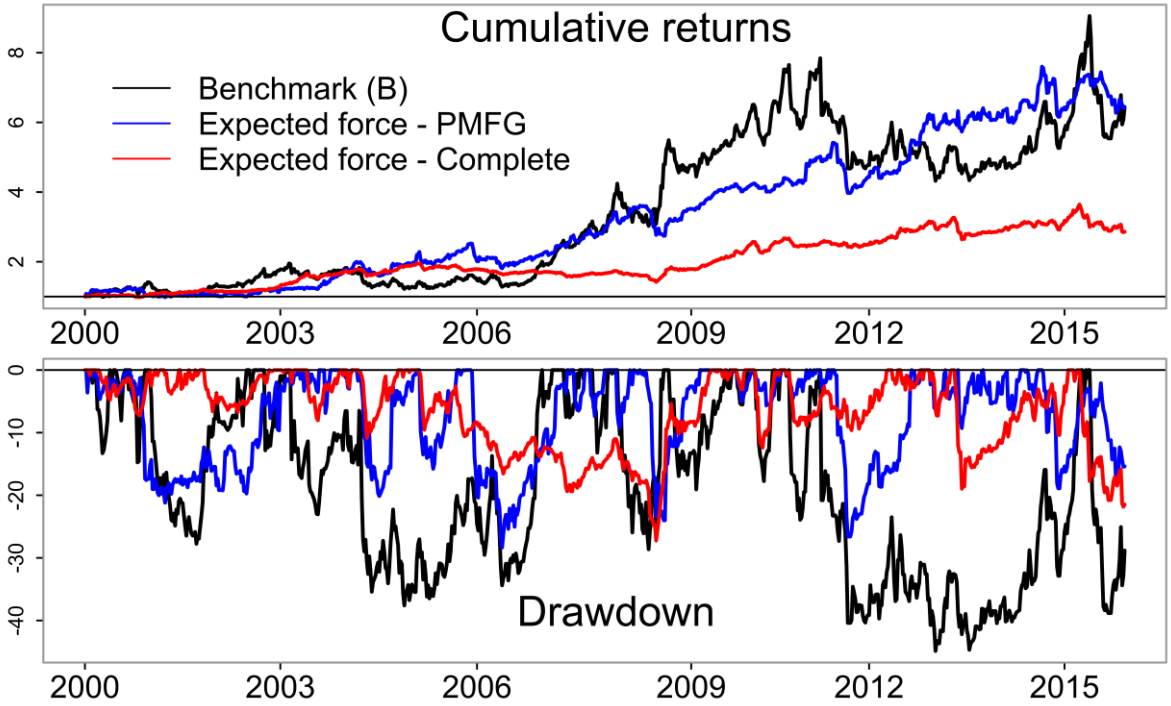


Figure 4 Cumulative returns and drawdowns of return maximization strategies

*Notes: We show the benchmark strategy and only two individual strategies that performed the best according to the SR, SR10, and SR5 portfolio performance measures.*

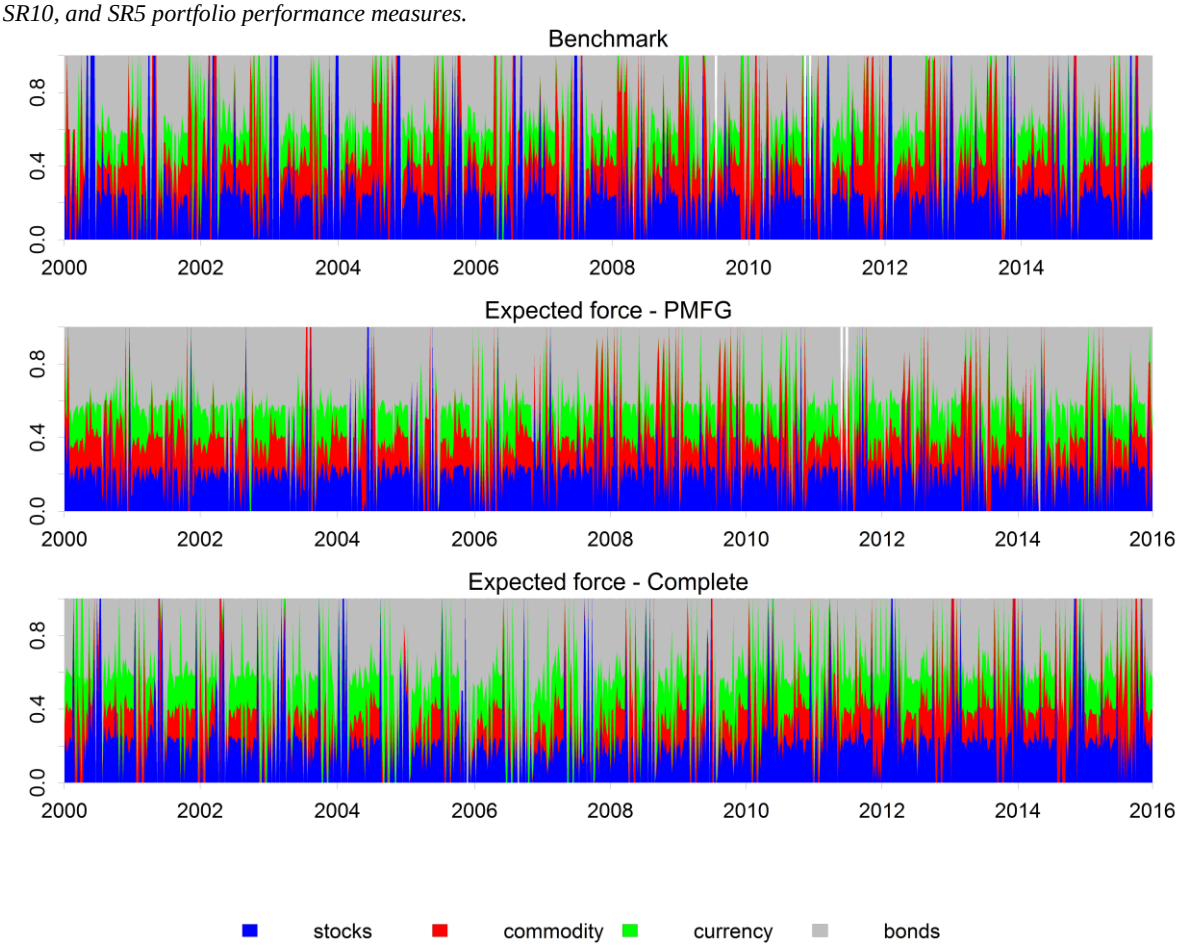


Figure 5 Asset allocations for selected return maximization strategies

Table 2 Return maximization portfolio optimization strategies

| | Portfolio return characteristics | | | | | | | | | | | | Performance measures Hansen et al.'s (2011) test | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **Min** | **Q1** | **Q3** | **Max** | **SD** | **DD** | **ES10** | **ES5** | **ES1** | **BR** | **Turnover** | **BETC** | **M** | **SR** | **SR10** | **SR5** | **SR1** |
| V00: Benchmark (B) | -20.793 | -1.471 | 2.337 | 16.558 | 3.607 | -0.189 | -0.067 | -0.086 | -0.136 | 0.123 | 0.364 | 0.0076 | 0.287 | 0.080 | 0.043 | 0.033 | 0.021 |
| ***Individual strategies*** | | | | | | | | | | | | | | | | | |
| *Betweenness centrality* | | | | | | | | | | | | | | | | | |
| V01: MST | -14.748 | -0.570 | 0.985 | 11.876 | 2.115 | -0.066 | -0.040 | -0.057 | -0.102 | 0.112 | 1.177 | 0.0013 | 0.140 | 0.066 | 0.035 | 0.025 | 0.014 |
| V02: PMFG | -14.901 | -0.651 | 1.041 | 15.879 | 2.292 | -0.118 | -0.043 | -0.057 | -0.100 | 0.057 | 1.363 | 0.0007 | 0.095 | 0.041 | 0.022 | 0.016 | 0.009 |
| V03: Threshold | -14.748 | -0.552 | 1.089 | 11.676 | 2.211 | -0.118 | -0.039 | -0.053 | -0.087 | 0.176 | 1.534 | 0.0013 | 0.202 | 0.091 | 0.052 | 0.038 | 0.023 |
| *Eigenvalue centrality* | | | | | | | | | | | | | | | | | |
| V04: Complete | -16.443 | -0.658 | 0.847 | 9.181 | 1.970 | -0.095 | -0.034 | -0.048 | -0.093 | 0.089 | 1.126 | 0.0010 | 0.117 | 0.060 | 0.034 | 0.024 | 0.013 |
| V05: MST | -14.326 | -0.555 | 0.886 | 32.593 | 2.433 | -0.208 | -0.042 | -0.057 | -0.096 | 0.045 | 1.459 | 0.0005 | 0.077 | 0.017 | 0.018 | 0.014 | 0.008 |
| V06: PMFG | -16.010 | -0.515 | 1.039 | 12.492 | 2.285 | -0.095 | -0.039 | -0.056 | -0.112 | 0.165 | 1.517 | 0.0013 | 0.201 | 0.088 | 0.051 | 0.036 | 0.018 |
| V07: Threshold | -16.443 | -0.643 | 0.915 | 8.561 | 2.147 | -0.193 | -0.043 | -0.060 | -0.101 | -0.001 | 1.510 | 0.0001 | 0.023† | 0.011 | 0.005 | 0.004 | 0.002 |
| *Expected force centrality* | | | | | | | | | | | | | | | | | |
| V08: Complete | -7.571 | -0.463 | 0.909 | 8.229 | 1.390 | -0.067 | -0.026 | -0.034 | -0.056 | 0.165 | 0.151 | 0.0089 | 0.137 | 0.099 | 0.053 | 0.041 | 0.024 |
| V09: MST | -25.315 | -0.624 | 0.997 | 24.777 | 2.903 | -0.152 | -0.052 | -0.075 | -0.137 | 0.024 | 1.521 | 0.0005 | 0.079 | 0.027 | 0.015 | 0.010 | 0.006 |
| V10: PMFG | -9.646 | -0.591 | 1.159 | 12.348 | 2.371 | -0.077 | -0.043 | -0.059 | -0.087 | 0.198 | 1.504 | 0.0016 | 0.255 | 0.108 | 0.060 | 0.043 | 0.029 |
| V11: Threshold | -20.438 | -0.586 | 1.016 | 18.695 | 2.576 | -0.144 | -0.046 | -0.066 | -0.134 | 0.056 | 1.463 | 0.0007 | 0.105 | 0.041 | 0.023 | 0.016 | 0.008 |
| ***Combination strategies*** | | | | | | | | | | | | | | | | | |
| V12: Betweenness + B | -11.712 | -0.852 | 1.466 | 9.556 | 2.254 | -0.092 | -0.042 | -0.054 | -0.085 | 0.170 | 0.701 | 0.0030 | 0.215 | **0.095** | **0.052** | **0.040** | **0.025** |
| V13: Eigenvalue + B | -10.446 | -0.898 | 1.451 | 10.084 | 2.207 | -0.093 | -0.040 | -0.053 | -0.077 | 0.155 | 0.704 | 0.0027 | 0.196 | **0.089** | **0.048** | **0.037** | **0.025** |
| V14: Exp. force + B | -12.515 | -0.836 | 1.485 | 9.244 | 2.294 | -0.096 | -0.043 | -0.056 | -0.088 | 0.165 | 0.628 | 0.0030 | 0.215 | **0.094** | **0.051** | **0.038** | **0.025** |
| V15: MST + B | -13.122 | -0.859 | 1.525 | 11.903 | 2.370 | -0.102 | -0.044 | -0.058 | -0.090 | 0.141 | 0.739 | 0.0025 | 0.193 | 0.081 | 0.044 | 0.033 | 0.021 |
| V16: PMFG + B | -11.402 | -0.868 | 1.482 | 10.813 | 2.311 | -0.089 | -0.042 | -0.054 | -0.081 | 0.184 | 0.757 | 0.0029 | 0.235 | **0.102** | **0.057** | **0.044** | **0.029** |
| V17: Threshold + B | -11.325 | -0.950 | 1.460 | 7.739 | 2.262 | -0.104 | -0.042 | -0.055 | -0.085 | 0.150 | 0.764 | 0.0025 | 0.198 | 0.088 | 0.047 | 0.036 | 0.023 |
| V18: Complete + B | -10.382 | -0.820 | 1.470 | 9.484 | 2.151 | -0.087 | -0.039 | -0.052 | -0.079 | 0.168 | 0.392 | 0.0051 | 0.209 | **0.097** | **0.053** | **0.040** | **0.026** |

*Notes: Q1 and Q3 denote the first and third quartiles of portfolio returns. M is the average weekly return. SD is the standard deviation of weekly portfolio returns. DD is the average drawdown. ES is the expected shortfall calculated via the Cornish and Fisher (1938) expansion, while the number next to ES denotes the percentile of interest. BR is the Burke ratio. SR denotes the Sharpe ratio where in the denominator, the standard deviation was employed. SR10, SR5 and SR1 denote the modified Sharpe ratios, where the denominator is substituted by the absolute value of the expected shortfall as calculated before. Bolded values in the columns (M, SR, SR10, SR5, and SR1) denote that the given row model out-performs the benchmark model in a direct comparison at least at the 5% statistical significance as indicated by the Hansen* et al.*'s (2011) test. Only one strategy (V07) is out-performed by the benchmark model in a direct comparison at least at the 5% statistical significance (indicated by †).*

### 4.2 Evaluation of risk minimization strategies

Our second approach was based on risk minimization portfolio strategies. Overall, risk minimization network-based asset allocation strategies led to even better risk-return outcomes. The related descriptive statistics are reported in Table 3, and they clearly show that while the benchmark portfolio (*R*00) shows the lowest levels of variability in portfolio returns as measured by *SD*, the return is also the smallest. However, the tradeoff between the two was found to be much better for individual network-based portfolio strategies, as on average the *SD* values of portfolio returns increased by 88.5% while average portfolio returns increased 188.4% on average. This is confirmed by the Sharpe ratios, which are largely significantly improved relative to the benchmark model. In fact, the Sharpe ratios are improved by approximately 51.7% on average for individual strategies while they are improved by 65.7% for combination strategies.

As noted above, we find that the network-based asset allocation strategies improve the risk-return profile of the benchmark portfolio. Although the absolute value of the left-tail risk is almost always larger as measured by the average drawdown (*DD*) and expected shortfall (*ES*), both the Burke ratio (*BR*) and modified Sharpe ratios (*SR10*, *SR05*, *SR1*) show that this additional risk is more than adequately compensated for by increased portfolio returns.

Figure 6 shows the striking difference between the benchmark strategy and network-based strategies: the returns are much smaller for the benchmark model relative to the selected set of trading strategies (the best ones were selected according to *SR*, *SR*5, and *SR*10). Consequently, the drawdown is also smaller. As is shown in Figure 6, we found that the greatest peak in the drawdown occurred during the financial crisis, which spurred the sub-par performance of all portfolios.

Time varying asset allocation is visualized in Figure 7. With the exception of the benchmark portfolio, there appear to be short periods during which all investor wealth was placed into bonds. Additional observations show that shares allocated to different asset classes appear to differ by strategy. For example, the "Expected force-Threshold" (*R*11) strategy appears to have the largest share in the bond, while "Expected force-Complete" (*R*08) strategy allocates more to much riskier stocks.

One might question whether the improved risk-return portfolio characteristics are offset by increased transaction costs. In fact, the turnover is larger for all individual network-based asset allocation strategies except for that of one strategy (*R*08). However, the increased turnover seems to be more than compensated for by increased returns, as the BETC is larger for all individual and combination network-based strategies than the BETC of the benchmark. We therefore conclude that the improved risk-return profile of network-based asset allocation strategies is robust with respect to transaction costs.

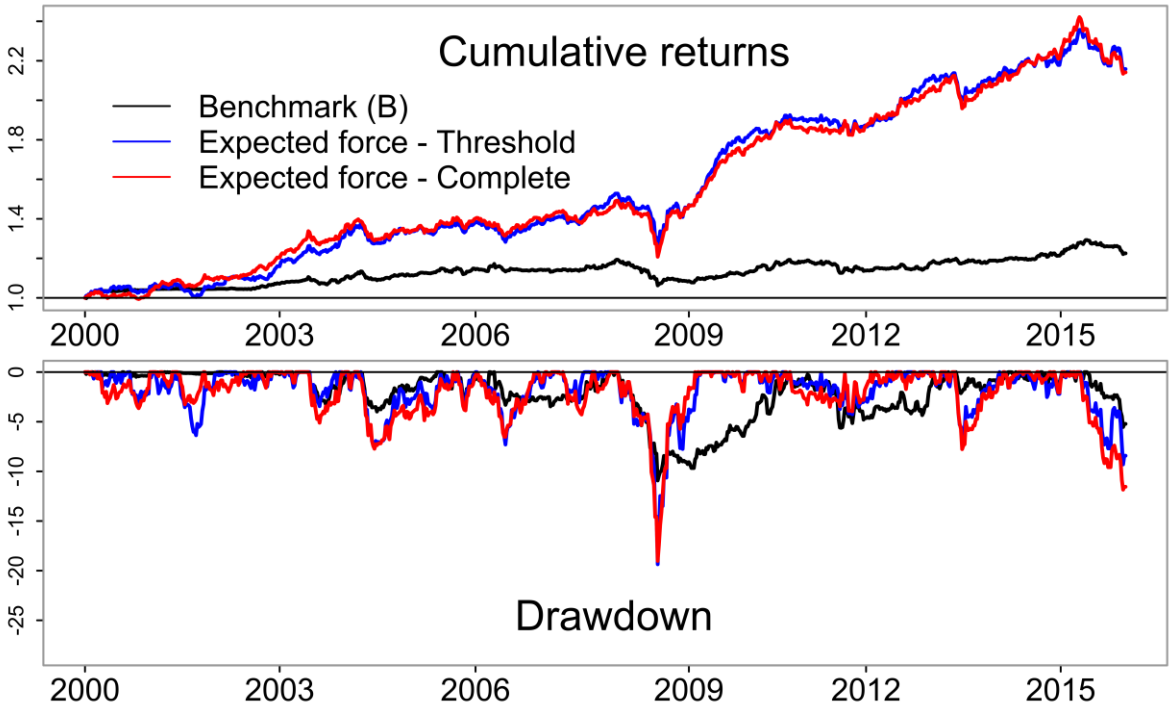


Figure 6 Cumulative returns and drawdowns of risk minimization strategies

*Notes: We show the benchmark strategy and only two individual strategies that performed the best according to the SR, SR10, and SR5 portfolio performance measures.*

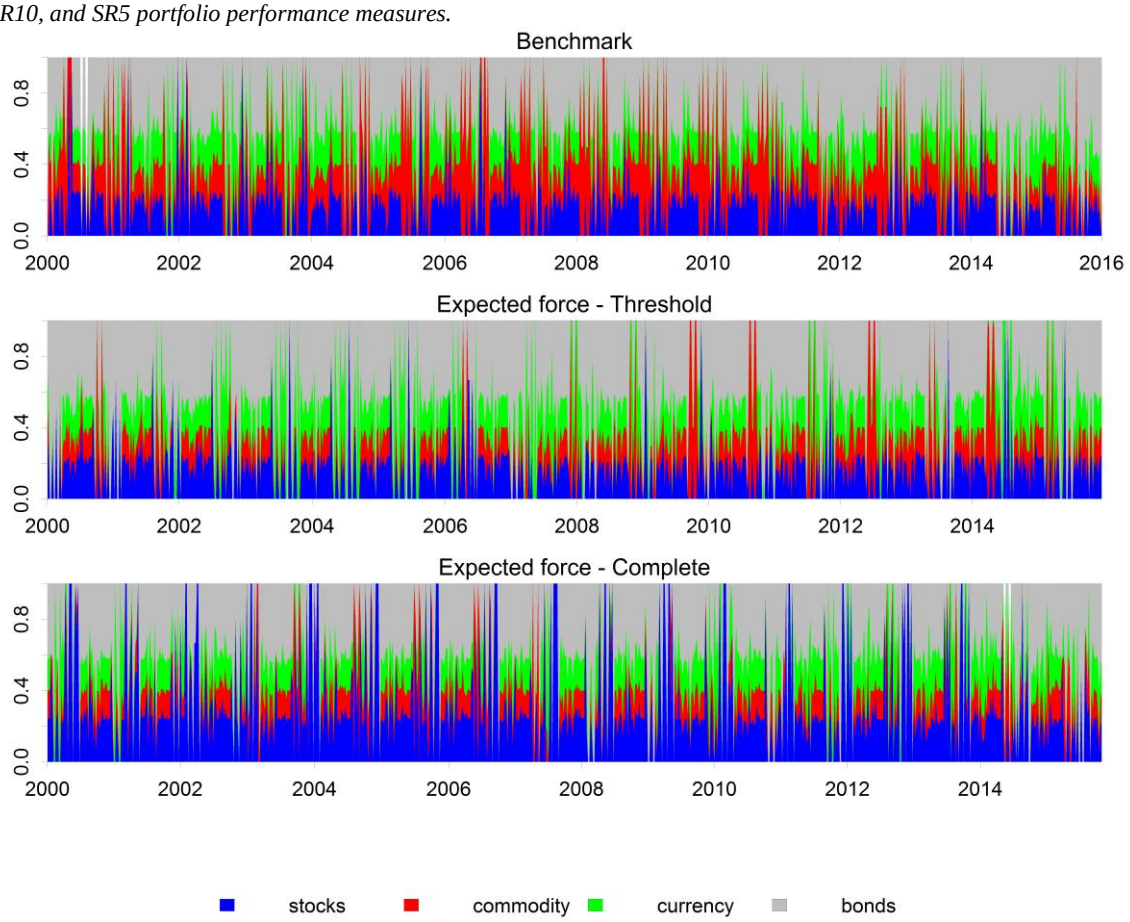


Figure 7 Asset allocations for selected risk minimization strategies

Table 3 Risk minimization portfolio optimization strategies

| | Portfolio return characteristics | | | | | | | | | | | | Performance measures Hansen et al.'s (2011) test | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **Min** | **Q1** | **Q3** | **Max** | **SD** | **DD** | **ES10** | **ES5** | **ES1** | **BR** | **Turnover** | **BETC** | **M** | **SR** | **SR10** | **SR5** | **SR1** |
| R00: Benchmark (B) | -2.949 | -0.125 | 0.220 | 1.906 | 0.428 | -0.018 | -0.008 | -0.011 | -0.016 | 0.133 | 0.352 | 0.0007 | 0.029 | 0.068 | 0.037 | 0.027 | 0.018 |
| ***Individual strategies*** | | | | | | | | | | | | | | | | | |
| *Betweenness centrality* | | | | | | | | | | | | | | | | | |
| R01: MST | -7.367 | -0.274 | 0.516 | 4.491 | 0.844 | -0.023 | -0.014 | -0.019 | -0.032 | 0.200 | 0.499 | 0.0018 | **0.093** | **0.110** | **0.066** | **0.049** | **0.029** |
| R02: PMFG | -8.506 | -0.294 | 0.548 | 5.437 | 0.873 | -0.026 | -0.014 | -0.019 | -0.035 | 0.179 | 0.502 | 0.0018 | **0.088** | **0.101** | **0.062** | **0.045** | **0.025** |
| R03: Threshold | -4.041 | -0.316 | 0.509 | 4.414 | 0.824 | -0.025 | -0.014 | -0.018 | -0.030 | 0.164 | 0.754 | 0.0009 | **0.077** | **0.094** | **0.054** | **0.042** | **0.025** |
| *Eigenvalue centrality* | | | | | | | | | | | | | | | | | |
| R04: Complete | -5.987 | -0.288 | 0.499 | 5.172 | 0.762 | -0.022 | -0.013 | -0.016 | -0.026 | 0.232 | 0.580 | 0.0015 | **0.092** | **0.120** | **0.072** | **0.056** | **0.035** |
| R05: MST | -5.352 | -0.274 | 0.463 | 4.984 | 0.771 | -0.040 | -0.014 | -0.018 | -0.031 | 0.134 | 0.747 | 0.0008 | **0.059** | **0.071** | **0.043** | **0.033** | **0.019** |
| R06: PMFG | -5.079 | -0.293 | 0.485 | 3.911 | 0.785 | -0.022 | -0.013 | -0.017 | -0.028 | 0.223 | 0.727 | 0.0012 | **0.089** | **0.114** | **0.068** | **0.052** | **0.032** |
| R07: Threshold | -6.368 | -0.305 | 0.471 | 4.173 | 0.744 | -0.026 | -0.013 | -0.017 | -0.026 | 0.163 | 0.824 | 0.0008 | **0.068** | **0.091** | **0.053** | **0.041** | **0.026** |
| *Expected force centrality* | | | | | | | | | | | | | | | | | |
| R08: Complete | -5.170 | -0.258 | 0.505 | 4.923 | 0.758 | -0.022 | -0.013 | -0.017 | -0.028 | 0.218 | 0.054 | 0.0170 | **0.096** | **0.126** | **0.074** | **0.057** | **0.034** |
| R09: MST | -8.728 | -0.283 | 0.498 | 4.403 | 0.843 | -0.029 | -0.014 | -0.019 | -0.033 | 0.150 | 0.742 | 0.0008 | **0.068** | **0.081** | **0.047** | **0.037** | **0.020** |
| R10: PMFG | -6.389 | -0.333 | 0.501 | 6.583 | 0.853 | -0.025 | -0.014 | -0.019 | -0.031 | 0.189 | 0.759 | 0.0012 | **0.092** | **0.107** | **0.066** | **0.049** | **0.030** |
| R11: Threshold | -5.616 | -0.281 | 0.499 | 5.416 | 0.820 | -0.019 | -0.014 | -0.018 | -0.030 | 0.226 | 0.770 | 0.0012 | **0.098** | **0.120** | **0.072** | **0.055** | **0.033** |
| ***Combination strategies*** | | | | | | | | | | | | | | | | | |
| R12: Betweenness + B | -6.271 | -0.267 | 0.498 | 4.592 | 0.779 | -0.022 | -0.013 | -0.018 | -0.030 | 0.202 | 0.335 | 0.0016 | **0.088** | **0.114** | **0.067** | **0.050** | **0.030** |
| R13: Eigenvalue + B | -5.697 | -0.258 | 0.428 | 3.495 | 0.697 | -0.023 | -0.012 | -0.016 | -0.025 | 0.200 | 0.347 | 0.0015 | **0.077** | **0.111** | **0.064** | **0.049** | **0.030** |
| R14: Exp. force + B | -6.476 | -0.275 | 0.475 | 5.175 | 0.761 | -0.021 | -0.012 | -0.017 | -0.029 | 0.207 | 0.313 | 0.0018 | **0.089** | **0.116** | **0.071** | **0.053** | **0.031** |
| R15: MST + B | -7.149 | -0.264 | 0.466 | 4.418 | 0.760 | -0.025 | -0.013 | -0.017 | -0.030 | 0.171 | 0.357 | 0.0014 | **0.074** | **0.097** | **0.056** | **0.042** | **0.024** |
| R16: PMFG + B | -6.658 | -0.294 | 0.480 | 4.928 | 0.791 | -0.021 | -0.013 | -0.017 | -0.030 | 0.208 | 0.354 | 0.0016 | **0.090** | **0.113** | **0.069** | **0.052** | **0.030** |
| R17: Threshold + B | -5.342 | -0.262 | 0.470 | 4.057 | 0.739 | -0.022 | -0.013 | -0.017 | -0.026 | 0.198 | 0.384 | 0.0013 | **0.081** | **0.110** | **0.065** | **0.049** | **0.031** |
| R18: Complete + B | -5.443 | -0.254 | 0.481 | 5.006 | 0.737 | -0.021 | -0.012 | -0.016 | -0.027 | 0.225 | 0.248 | 0.0024 | **0.094** | **0.128** | **0.076** | **0.057** | **0.035** |

*Notes: Q1 and Q3 denote the first and third quartiles of portfolio returns. M is the average weekly return. SD is the standard deviation of weekly portfolio returns. DD is the average drawdown. ES is the expected shortfall calculated via the Cornish and Fisher (1938) expansion, while the number next to ES denotes the percentile of interest. BR is the Burke ratio. SR denotes the Sharpe ratio where in the denominator, the standard deviation was employed. SR10, SR5 and SR1 denote the modified Sharpe ratios, where the denominator is substituted by the absolute value of the expected shortfall as calculated before. Bolded values in the columns (M, SR, SR10, SR5, and SR1) denote that the given row model out-performs the benchmark model in a direct comparison at least at the 5% statistical significance as indicated by the Hansen et al.'s (2011) test. Symbol † in the columns (M, SR, SR10, SR5, and SR1) denote that the given row model is out-performed by the benchmark model in a direct comparison at least at the 5% statistical significance as indicated by the Hansen* et al.*'s (2011) test. No strategy is out-performed by the benchmark model in a direct comparison at least at the 5% statistical significance.*

## Concluding remarks

We contribute to the emerging literature on network applications in finance and asset allocation (e.g., Onnela et al., 2003; Kaya, 2015; Baitinger and Papenbrock, 2016; López de Prado, 2016; Peralta and Zareii, 2016; Huang et al., 2018, Xidonas et al., 2017; Khin et al., 2017) by proposing simple network-based asset allocation extensions of standard Markowitz portfolio strategies. Return maximization and risk minimization portfolios are adjusted by adding only one constraint that states that asset weights must be ordered in the same way as the reciprocal of asset centrality in a given network. We consider four types of correlation-based networks: complete graphs, the minimum spanning tree, planar maximally filtered graphs, and threshold significance graphs. We further consider three centrality measures: betweenness, eigenvalue centrality, and the expected force.

We use data from 1999 to 2015 for 45 different assets. We compare risk-return characteristics in an out-of-sample framework of all network-based asset allocation strategies with their respective benchmark models. Within our sample, we show that utilizing information on the interconnectedness of various asset returns given the topological structure of a network improves the risk-return characteristics of standard benchmark portfolios. It is quite clear that the constructed network takes into account complex relationships between assets beyond those measured by correlations.

**Literature**


[1] Andersen, T. G., Bollerslev, T., Diebold, F. X., Vega, C. (2007). Real-time price discovery in global stock, bond and foreign exchange markets. *Journal of international Economics*, 73(2), 251-277.

[2] Andrews, D. W. (1991). Heteroskedasticity and autocorrelation consistent covariance matrix estimation. *Econometrica*, 59(3), 817-858.

[3] Baele, L., Bekaert, G., Inghelbrecht, K. (2010). The determinants of stock and bond return comovements. *Review of Financial Studies*, 23(6), 2374-2428.

[4] Baitinger, E., Papenbrock, J. (2016). Interconnectedness Risk and Active Portfolio Management. *Available at SSRN 2796443*.

[5] Baruník, J., Kočenda, E., Vácha, L. (2015). Volatility spillovers across petroleum markets. *Energy Journal*, 36(3), 309–329.

[6] Baruník, J., Kočenda, E., Vácha, L. (2016). Asymmetric connectedness of stocks: How does bad and good volatility spill over the U.S. stock market? *Journal of Financial Markets*, 27, 55-78.

[7] Bates, J.M., Granger, C.W.J. (1969). The combination of forecasts. *Operational Research Quarterly*, 20(4), 451-468.

[8] Baumöhl, E., Lyócsa, Š. (2017). Directional predictability from stock market sector indices to gold: A cross-quantilogram analysis. *Finance Research Letters* 23, 152–164.

[9] Baur, D. G., Lucey, B. M. (2010). Is gold a hedge or a safe haven? An analysis of stocks, bonds and gold. *Financial Review*, *45*(2), 217-229.

[10] Billio, M., Getmansky, M., Lo, A. W., Pelizzon, L. (2012). Econometric measures of connectedness and systemic risk in the finance and insurance sectors. *Journal of Financial Economics*, 104(3), 535-559.

[11] Black, F., Litterman, R. B. (1991). Asset allocation: combining investor views with market equilibrium. *The Journal of Fixed Income*, 1(2), 7-18.

[12] Ciner, C., Gurdgiev, C., Lucey, B. M. (2013). Hedges and safe havens: An examination of stocks, bonds, gold, oil and exchange rates. *International Review of Financial Analysis*, *29*, 202-211.

[13] Cornish, E. A., Fisher, R. A. (1938). Moments and Cumulants in the Specification of Distributions. *Review of the International Statistical Institute*, 5(4), 307-320.

[14] Cunado, J., de Gracia, F. P. (2014). Oil price shocks and stock market returns: Evidence for some European countries. *Energy Economics*, 42, 365-377.

[15] de Athayde, G. M., Flôres, R. G. (2004). Finding a maximum skewness portfolio—a general solution to three-moments portfolio choice. *Journal of Economic Dynamics and Control*, 28(7), 1335-1352.

[16] DeMiguel, V., Garlappi, L., Uppal, R. (2007). Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy?. The review of Financial studies, 22(5), 1915-1953.

[17] Diebold, F. X., Yilmaz, K. (2014). On the network topology of variance decompositions: Measuring the connectedness of financial firms. *Journal of Econometrics*, 182(1), 119-134.

[18] Diebold, F. X., Yilmaz, K. (2015). *Financial and Macroeconomic Connectedness: A Network Approach to Measurement and Monitoring*. Oxford University Press, Oxford.

[19] Ehrmann, M., Fratzscher, M., Rigobon, R. (2011). Stocks, bonds, money markets and exchange rates: measuring international financial transmission. *Journal of Applied Econometrics*, 26(6), 948-974.

[20] Fatum, R., Yamamoto, Y. (2016). Intra-safe haven currency behavior during the global financial crisis. *Journal of International Money and Finance*, 66, 49-64.

[21] Flavin, T. J., Morley, C. E., Panopoulou, E. (2014). Identifying safe haven assets for equity investors through an analysis of the stability of shock transmission. *Journal of International Financial Markets, Institutions and Money*, 33, 137-154.

[22] Grisse, C., Nitschka, T. (2015). On financial risk and the safe haven characteristics of Swiss franc exchange rates. *Journal of Empirical Finance*, 32, 153-164.

[23] Hansen, P. R., Lunde, A., Nason, J. M. (2011). The Model Confidence Set. *Econometrica*, 79(2), 453-497.

[24] Huang, W., Lai, P.-C., Bessler, D. A. (2018). On the changing structure among Chinese equity markets: Hong Kong, Shanghai, and Shenzhen. *European Journal of Operational Research*, 264(3), 1020-1032.

[25] Kane, A. (1982). Skewness Preference and Portfolio Choice, *Journal of Financial and Quantitative Analysis*, 17(1), 15-25.

[26] Kaya, H. (2015). Eccentricity in asset management. *Network Theory in Finance*, 1(1), 1-32.

[27] Khin T. L., Qu, R., MacCarthy, B.L. (2017). Mean-VaR portfolio optimization: A nonparametric approach, *European Journal of Operational Research*, 260(2), 751-766.

[28] Krugman, P. R. (1980). Oil and the Dollar. *NBER Working Paper*, (w0554).

[29] Kruskal, J. B. (1956). On the shortest spanning subtree of a graph and the traveling salesman problem. *Proceedings of the American Mathematical society*, 7(1), 48-50.

[30] Lawyer, G. (2015). Understanding the influence of all nodes in a network. *Scientific Reports*, *5*, no. 8665.

[31] López de Prado, M. (2016). Building diversified portfolios that outperform out-of-sample. *The Journal of Portfolio Management*, 42(4), 59-69.

[32] Mantegna, R. N. (1999). Hierarchical structure in financial markets. *The European Physical Journal B-Condensed Matter and Complex Systems*, 11(1), 193-197.

[33] Mantegna, R. N., Stanley, H. E. (1999). *Introduction to econophysics: correlations and complexity in finance*. Cambridge university press.

[34] Markowitz, H. (1952). Portfolio selection. *The Journal of Finance*, 7(1), 77-91.

[35] Markowitz, H. (1956). The optimization of a quadratic function subject to linear constraints. *Naval research logistics*, 3(1-2), 111-133.

[36] Mensi, W., Beljid, M., Boubaker, A., Managi, S. (2013). Correlations and volatility spillovers across commodity and stock markets: Linking energies, food, and gold. *Economic Modelling*, 32, 15–22.

[37] Nazlioglu, S., Erdem, C., Soytas, U. (2013). Volatility spillover between oil and agricultural commodity markets. *Energy Economics*, 36, 658–665.

[38] Onnela, J. P., Chakraborti, A., Kaski, K., Kertesz, J., Kanto, A. (2003). Dynamics of market correlations: Taxonomy and portfolio analysis. *Physical Review E*, 68(5), 056110.

[39] Panopoulou, E., Pittis, N., Kalyvitis, S. (2010). Looking far in the past: revisiting the growth-returns nexus with non-parametric tests. *Empirical Economics*, 38(3), 743-766.

[40] Pantaleo, E., Tumminello, M., Lillo, F., Mantegna, R. N. (2011). When do improved covariance matrix estimators enhance portfolio optimization? An empirical comparative study of nine estimators. *Quantitative Finance*, 11(7), 1067-1080.

[41] Park, J., Ratti, R. A. (2008). Oil price shocks and stock markets in the US and 13 European countries. *Energy economics*, 30(5), 2587-2608.

[42] Peralta, G., Zareei, A. (2016). A network approach to portfolio selection. *Journal of Empirical Finance*, 38, 157-180.

[43] Reboredo, J. C. (2013). Is gold a hedge or safe haven against oil price movements. *Resources Policy*, 38(2), 130-137.

[44] Reboredo, J. C., Rivera-Castro, M. A., Zebende, G. F. (2014). Oil and US dollar exchange rate dependence: A detrended cross-correlation approach. *Energy Economics*, 42, 132-139.

[45] Rom, B. M., Ferguson, K. W. (1994). Post-modern portfolio theory comes of age. *The Journal of Investing*, 3(3), 11-17.

[46] Shiller, R. J., Beltratti, A. E. (1992). Stock prices and bond yields: Can their comovements be explained in terms of present value models? *Journal of Monetary Economics*, 30(1), 25-46.

[47] Tola, V., Lillo, F., Gallegati, M., Mantegna, R. N. (2008). Cluster analysis for portfolio optimization. *Journal of Economic Dynamics and Control*, 32(1), 235-258.

[48] Tse, Ch. K., Liu, J., Lau, F. C. M. (2010). A network perspective of stock market. *Journal of Empirical Finance*, 17(4), 659-667.

[49] Tumminello, M., Aste, T., Di Matteo, T., Mantegna, R. N. (2005). A tool for filtering information in complex systems. *Proceedings of the National Academy of Sciences of the United States of America*, 102(30), 10421-10426.

[50] Výrost, T., Lyócsa, Š., Baumöhl, E. (2015). Granger causality stock market networks: Temporal proximity and preferential attachment. *Physica A: Statistical Mechanics and its Applications*, 427, 262-276.

[51] Xidonas, P., Mavrotas, G., Hassapis, C., Zopounidis, C. (2017). Robust multiobjective portfolio optimization: A minimax regret approach, *European Journal of Operational Research* (in press).

## Appendix

Table A.1 Data descriptions

| Asset class | Abbreviation | Source |
|---|---|---|
| **Stocks** | | |
| Bovespa | BVSP | finance.yahoo |
| DAX 30 | DAX | finance.yahoo |
| FTSE 100 | FTSE100 | finance.yahoo |
| Kospi | KOSPI | finance.yahoo |
| Merval | MERV | finance.yahoo |
| Nikkei 225 | N225 | finance.yahoo |
| SMI 20 | SMI | finance.yahoo |
| S&P 500 | SP500 | finance.yahoo |
| SSE Composite | SSE | finance.yahoo |
| TSE 300 | TSE | finance.yahoo |
| TSEC | TWII | finance.yahoo |
| **Commodities** | | |
| Oil (Brent) | Brent | quandl.com |
| Cocoa | Cocoa | quandl.com |
| Copper | Copper | quandl.com |
| Cotton | Cotton | quandl.com |
| Gold | Gold | quandl.com |
| Natural gas | NatGas | quandl.com |
| Silver | Silver | quandl.com |
| **Currencies** | | |
| AUD/USD | AUD_USD | quandl.com |
| CAD/USD | CAD_USD | quandl.com |
| CHF/USD | CHF_USD | quandl.com |
| CNY/USD | CNY_USD | quandl.com |
| EUR/USD | EUR_USD | quandl.com |
| GBP/USD | GBP_USD | quandl.com |
| JPY/USD | JPY_USD | quandl.com |
| NOK/USD | NOK_USD | quandl.com |
| **Money/bond market** | | |
| Moody's Aaa US corporate bonds | bAAA | fred.stlouisfed.org |
| Moody's Bbb US corporate bonds | bBBB | fred.stlouisfed.org |
| U.S. Commercial Paper. Financial. (1-month) | bCPF_1M | fred.stlouisfed.org |
| U.S. Commercial Paper. Non-Financial. (1-month) | bCPNF_1M | fred.stlouisfed.org |
| EMEA Corporate Bond Total Return Index | bEMEA_corp | fred.stlouisfed.org |
| Emerging Markets Corporate Bond Index | bEMER_corp | fred.stlouisfed.org |
| Emerging Markets High Grade Corporate Bond Index | bEMER_corp_high | fred.stlouisfed.org |
| Euro Emerging Markets Corporate Bond | bEMER_EURO_corp | fred.stlouisfed.org |
| Euro High Yield Bonds | bEUR_HY | fred.stlouisfed.org |
| German Government Bonds (1 year) | bGER_1Y | quandl.com |
| German Government Bonds (5 year) | bGER_5Y | quandl.com |
| German Government Bonds (20 year) | bGER_20Y | quandl.com |
| German Corporate Bonds | bGER_corp | bundesbank.de |
| Japanese Government Bonds (1 year) | bJPN_1Y | mof.go.jp |
| Japanese Government Bonds (5 year) | bJPN_5Y | mof.go.jp |
| Japanese Government Bonds (20 year) | bJPN_20Y | mof.go.jp |
| U.S. Treasury Constant Maturity (1 year) | bUS_1Y | fred.stlouisfed.org |
| U.S. Treasury Constant Maturity (5 year) | bUS_5Y | fred.stlouisfed.org |
| U.S. Treasury Constant Maturity (20 year) | bUS_20Y | fred.stlouisfed.org |

Table A.2 Return maximization portfolio optimization strategies evaluated against risk-free rate (3-month U.S. Treasury bill)

| | Portfolio return characteristics | | | | | | | | | | | | Performance measures Hansen et al.'s (2011) test | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **Min** | **Q1** | **Q3** | **Max** | **SD** | **DD** | **ES10** | **ES5** | **ES1** | **BR** | **Turnover** | **BETC** | **M** | **SR** | **SR10** | **SR5** | **SR1** |
| R00: Benchmark (B) | -20.794 | -1.488 | 2.313 | 16.556 | 3.606 | -0.189 | -0.067 | -0.086 | -0.136 | 0.100 | 0.364 | 0.0076 | 0.255 | 0.071 | 0.038 | 0.030 | 0.019 |
| ***Individual strategies*** | | | | | | | | | | | | | | | | | |
| *Betweenness centrality* | | | | | | | | | | | | | | | | | |
| R01: MST | -14.748 | -0.602 | 0.945 | 11.872 | 2.115 | -0.066 | -0.040 | -0.057 | -0.102 | 0.076 | 1.177 | 0.0013 | 0.108 | 0.051 | 0.027 | 0.019 | 0.011 |
| R02: PMFG | -14.901 | -0.693 | 1.014 | 15.878 | 2.293 | -0.118 | -0.043 | -0.058 | -0.100 | 0.027 | 1.363 | 0.0007 | 0.062 | 0.027 | 0.015 | 0.011 | 0.006 |
| R03: Threshold | -14.748 | -0.583 | 1.052 | 11.676 | 2.210 | -0.118 | -0.039 | -0.053 | -0.087 | 0.135 | 1.534 | 0.0013 | 0.169 | 0.077 | 0.043 | 0.032 | 0.019 |
| *Eigenvalue centrality* | | | | | | | | | | | | | | | | | |
| R04: Complete | -16.444 | -0.694 | 0.816 | 9.181 | 1.971 | -0.095 | -0.035 | -0.048 | -0.094 | 0.056 | 1.126 | 0.0010 | 0.085 | 0.043 | 0.025 | 0.018 | 0.009 |
| R05: MST | -14.416 | -0.615 | 0.848 | 32.593 | 2.434 | -0.208 | -0.042 | -0.057 | -0.096 | 0.013 | 1.459 | 0.0005 | 0.044 | 0.002 | 0.011 | 0.008 | 0.005 |
| R06: PMFG | -16.051 | -0.547 | 0.998 | 12.470 | 2.286 | -0.095 | -0.040 | -0.057 | -0.113 | 0.131 | 1.517 | 0.0013 | 0.169 | 0.074 | 0.043 | 0.030 | 0.015 |
| R07: Threshold | -16.444 | -0.668 | 0.884 | 8.561 | 2.147 | -0.193 | -0.044 | -0.060 | -0.101 | -0.026 | 1.510 | 0.0001 | -0.009 | -0.004 | -0.002 | -0.002 | -0.001 |
| *Expected force centrality* | | | | | | | | | | | | | | | | | |
| R08: Complete | -7.572 | -0.507 | 0.877 | 8.228 | 1.392 | -0.067 | -0.026 | -0.034 | -0.056 | 0.114 | 0.151 | 0.0089 | 0.105 | 0.075 | 0.041 | 0.031 | 0.019 |
| R09: MST | -25.337 | -0.674 | 0.950 | 24.723 | 2.902 | -0.152 | -0.053 | -0.075 | -0.138 | 0.002 | 1.521 | 0.0005 | 0.046 | 0.016 | 0.009 | 0.006 | 0.003 |
| R10: PMFG | -9.697 | -0.623 | 1.152 | 12.316 | 2.372 | -0.077 | -0.043 | -0.059 | -0.087 | 0.160 | 1.504 | 0.0016 | 0.223 | 0.094 | 0.052 | 0.038 | 0.026 |
| R11: Threshold | -20.455 | -0.634 | 0.969 | 18.672 | 2.576 | -0.144 | -0.046 | -0.067 | -0.135 | 0.029 | 1.463 | 0.0007 | 0.073 | 0.028 | 0.016 | 0.011 | 0.005 |
| ***Combination strategies*** | | | | | | | | | | | | | | | | | |
| R12: Betweenness + B | -11.714 | -0.903 | 1.420 | 9.552 | 2.254 | -0.092 | -0.042 | -0.055 | -0.085 | 0.133 | 0.701 | 0.0030 | 0.183 | 0.081 | 0.044 | 0.034 | 0.022 |
| R13: Eigenvalue + B | -10.447 | -0.915 | 1.406 | 10.084 | 2.207 | -0.093 | -0.041 | -0.053 | -0.077 | 0.119 | 0.704 | 0.0027 | 0.163 | 0.074 | 0.040 | 0.031 | 0.021 |
| R14: Exp. force + B | -12.516 | -0.853 | 1.451 | 9.243 | 2.293 | -0.096 | -0.043 | -0.056 | -0.088 | 0.130 | 0.628 | 0.0030 | 0.183 | 0.080 | 0.043 | 0.033 | 0.021 |
| R15: MST + B | -13.124 | -0.873 | 1.494 | 11.903 | 2.369 | -0.102 | -0.044 | -0.058 | -0.090 | 0.108 | 0.739 | 0.0025 | 0.160 | 0.068 | 0.037 | 0.028 | 0.018 |
| R16: PMFG + B | -11.403 | -0.920 | 1.450 | 10.812 | 2.311 | -0.089 | -0.042 | -0.054 | -0.081 | 0.148 | 0.757 | 0.0029 | 0.203 | **0.088** | **0.049** | **0.038** | **0.025** |
| R17: Threshold + B | -11.326 | -0.985 | 1.419 | 7.738 | 2.261 | -0.104 | -0.042 | -0.055 | -0.085 | 0.114 | 0.764 | 0.0025 | 0.166 | 0.073 | 0.040 | 0.030 | 0.020 |
| R18: Complete + B | -10.383 | -0.847 | 1.417 | 9.483 | 2.151 | -0.087 | -0.040 | -0.052 | -0.079 | 0.132 | 0.392 | 0.0051 | 0.176 | 0.082 | 0.045 | 0.034 | 0.022 |

*Notes: Q1 and Q3 denote the first and third quartiles of portfolio returns. M is the average weekly return. SD is the standard deviation of weekly portfolio returns. DD is the average drawdown. ES is the expected shortfall calculated from the Cornish and Fisher (1938) expansion, while the number next to ES denotes the percentile of interest. BR is the Burke ratio. SR denotes the Sharpe ratio wherein the denominator standard deviation was employed. SR10, SR5 and SR1 denote the modified Sharpe ratios, where the denominator was substituted by the absolute value of the expected shortfall as calculated above. The bolded values in columns (M, SR, SR10, SR5, and SR1) denote at least 5% statistical significance against the benchmark value based on Hansen* et al.*'s (2011) test.*

Table A.3 Risk minimization portfolio optimization strategies evaluated against risk-free rate (3-month U.S. Treasury bill)

| | Portfolio return characteristics | | | | | | | | | | | | Performance measures Hansen et al.'s (2011) test | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **Min** | **Q1** | **Q3** | **Max** | **SD** | **DD** | **ES10** | **ES5** | **ES1** | **BR** | **Turnover** | **BETC** | **M** | **SR** | **SR10** | **SR5** | **SR1** |
| R00: Benchmark (B) | -2.958 | -0.162 | 0.193 | 1.821 | 0.430 | -0.018 | -0.008 | -0.011 | -0.017 | -0.017 | 0.352 | 0.0007 | -0.003 | -0.007 | -0.004 | -0.003 | -0.002 |
| ***Individual strategies*** | | | | | | | | | | | | | | | | | |
| *Betweenness centrality* | | | | | | | | | | | | | | | | | |
| R01: MST | -7.376 | -0.319 | 0.476 | 4.491 | 0.846 | -0.023 | -0.014 | -0.019 | -0.033 | 0.107 | 0.499 | 0.0018 | **0.061** | **0.072** | **0.043** | **0.032** | **0.019** |
| R02: PMFG | -8.514 | -0.333 | 0.527 | 5.437 | 0.875 | -0.026 | -0.015 | -0.020 | -0.036 | 0.092 | 0.502 | 0.0018 | **0.056** | **0.064** | **0.039** | **0.029** | **0.016** |
| R03: Threshold | -4.049 | -0.359 | 0.473 | 4.413 | 0.824 | -0.025 | -0.015 | -0.019 | -0.030 | 0.081 | 0.754 | 0.0009 | **0.045** | **0.054** | **0.031** | **0.024** | **0.015** |
| *Eigenvalue centrality* | | | | | | | | | | | | | | | | | |
| R04: Complete | -5.996 | -0.331 | 0.468 | 5.172 | 0.764 | -0.022 | -0.013 | -0.017 | -0.026 | 0.135 | 0.580 | 0.0015 | **0.059** | **0.078** | **0.047** | **0.036** | **0.023** |
| R05: MST | -5.361 | -0.326 | 0.431 | 4.984 | 0.772 | -0.040 | -0.014 | -0.018 | -0.031 | 0.049 | 0.747 | 0.0008 | 0.027 | 0.028 | 0.020 | 0.015 | 0.009 |
| R06: PMFG | -5.088 | -0.339 | 0.454 | 3.891 | 0.788 | -0.022 | -0.014 | -0.018 | -0.028 | 0.127 | 0.727 | 0.0012 | **0.057** | **0.073** | **0.043** | **0.033** | **0.020** |
| R07: Threshold | -6.377 | -0.351 | 0.439 | 4.171 | 0.746 | -0.026 | -0.013 | -0.017 | -0.026 | 0.070 | 0.824 | 0.0008 | **0.035** | **0.047** | **0.028** | **0.021** | **0.014** |
| *Expected force centrality* | | | | | | | | | | | | | | | | | |
| R08: Complete | -5.179 | -0.314 | 0.465 | 4.922 | 0.760 | -0.022 | -0.013 | -0.017 | -0.028 | 0.121 | 0.054 | 0.0170 | **0.063** | **0.084** | **0.049** | **0.037** | **0.023** |
| R09: MST | -8.737 | -0.316 | 0.464 | 4.383 | 0.843 | -0.029 | -0.015 | -0.019 | -0.034 | 0.067 | 0.742 | 0.0008 | **0.036** | **0.043** | **0.025** | **0.019** | **0.011** |
| R10: PMFG | -6.398 | -0.371 | 0.463 | 6.583 | 0.854 | -0.025 | -0.014 | -0.019 | -0.031 | 0.102 | 0.759 | 0.0012 | **0.059** | **0.070** | **0.043** | **0.032** | **0.019** |
| R11: Threshold | -5.624 | -0.306 | 0.456 | 5.415 | 0.822 | -0.019 | -0.014 | -0.018 | -0.030 | 0.128 | 0.770 | 0.0012 | **0.066** | **0.080** | **0.048** | **0.037** | **0.022** |
| ***Combination strategies*** | | | | | | | | | | | | | | | | | |
| R12: Betweenness + B | -6.280 | -0.303 | 0.468 | 4.592 | 0.781 | -0.022 | -0.013 | -0.018 | -0.030 | 0.106 | 0.335 | 0.0016 | **0.056** | **0.072** | **0.043** | **0.032** | **0.019** |
| R13: Eigenvalue + B | -5.705 | -0.304 | 0.397 | 3.494 | 0.699 | -0.023 | -0.012 | -0.016 | -0.026 | 0.096 | 0.347 | 0.0015 | **0.045** | **0.064** | **0.037** | **0.028** | **0.018** |
| R14: Exp. force + B | -6.485 | -0.299 | 0.439 | 5.175 | 0.762 | -0.021 | -0.013 | -0.017 | -0.029 | 0.108 | 0.313 | 0.0018 | **0.056** | **0.074** | **0.045** | **0.034** | **0.019** |
| R15: MST + B | -7.158 | -0.285 | 0.416 | 4.418 | 0.761 | -0.025 | -0.013 | -0.018 | -0.030 | 0.077 | 0.357 | 0.0014 | **0.041** | **0.054** | **0.032** | **0.024** | **0.014** |
| R16: PMFG + B | -6.667 | -0.329 | 0.444 | 4.927 | 0.792 | -0.021 | -0.013 | -0.018 | -0.030 | 0.108 | 0.354 | 0.0016 | **0.057** | **0.072** | **0.044** | **0.033** | **0.019** |
| R17: Threshold + B | -5.350 | -0.298 | 0.437 | 4.055 | 0.740 | -0.022 | -0.013 | -0.017 | -0.026 | 0.099 | 0.384 | 0.0013 | **0.049** | **0.066** | **0.039** | **0.029** | **0.019** |
| R18: Complete + B | -5.451 | -0.302 | 0.453 | 5.005 | 0.739 | -0.021 | -0.013 | -0.017 | -0.027 | 0.124 | 0.248 | 0.0024 | **0.062** | **0.084** | **0.050** | **0.038** | **0.023** |

*Notes: Q1 and Q3 denote the first and third quartiles of portfolio returns. M is the average weekly return. SD is the standard deviation of weekly portfolio returns. DD is the average drawdown. ES is the expected shortfall calculated from the Cornish and Fisher (1938) expansion, while the number next to ES denotes the percentile of interest. BR is the Burke ratio. SR denotes the Sharpe ratio wherein the denominator standard deviation was employed. SR10, SR5 and SR1 denote the modified Sharpe ratios, where the denominator was substituted by the absolute value of the expected shortfall as calculated above. The bolded values in columns (M, SR, SR10, SR5, and SR1) denote at least 5% statistical significance against the benchmark value based on Hansen* et al.*'s (2011) test.*